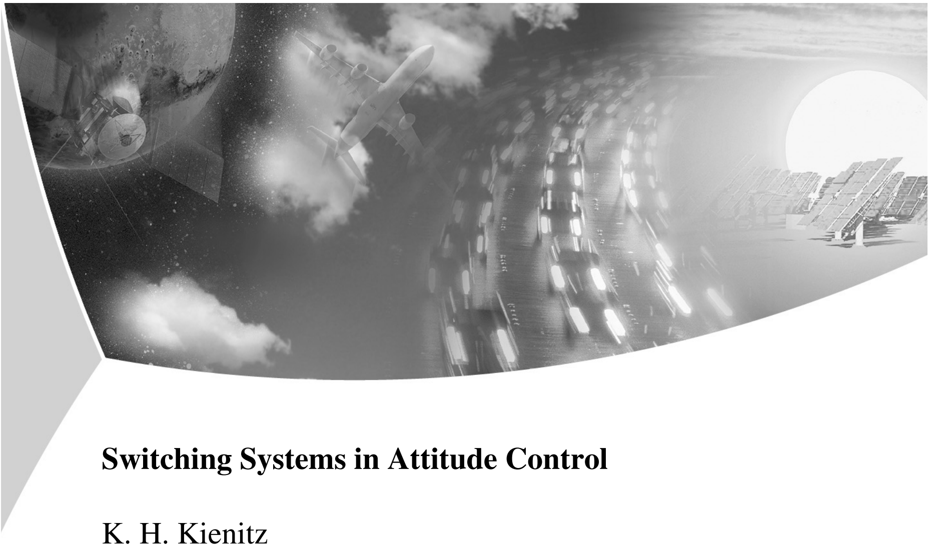

# Switching Systems in Attitude Control

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## Contents



- Acknowledgment:
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  - Neusa Maria Franco de Oliveira
  - Alexandre Rodrigues Mesquita

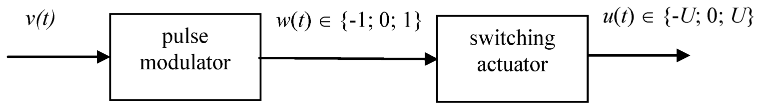


## 1. Introduction, motivation

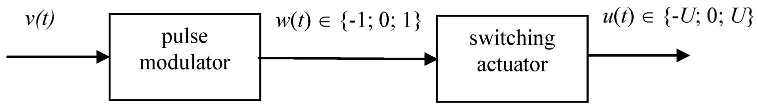
- Launch and space vehicles may use on-off thruster actuators for attitude control. Thrusters produce discontinuous control actions and are subject to switching constraints.
- This is an overview presentation that reports on ongoing work concerned with:
  - the analysis of system behavior when actuators are operated at the limits of their switching constraints;
  - a systematic handling of switching constraints.
- Goal: account for switching constraints at design time to ensure the best possible use of actuators.
- Pulse modulation and direct activation are considered.



## 2. Pulse modulation




- Output $u(t)$ takes values in $\{-U; 0; U\}$
- Switching command $w(t) \in \{-1; 0; 1\}$ is a function of the modulator input $v(t)$.
- In the baseline problem, actuator switching is subject to restrictions:
  - Minimum pulse duration (on-time): $t_{on}$
  - Minimum rest between successive pulses: $t_{off}$
- Provided both restrictions are satisfied, output will be $-U$ for negative actuator inputs, zero for zero input, and $U$ for positive inputs.
- Direct switching from $-U$ to $U$ (or vice-versa) is not allowed.




- A pulse modulator is needed that produces an actuator command signal, $w(t)$, such that:
  - It will not command a change of actuator output in violation of the actuator's switching restrictions;
  - For constant modulator input $v(t) = v_c$, the average value of $u(t)$ is $u_m = f(v_c)$, where $f: \Re \rightarrow [-U; U]$ is an arbitrary function chosen by the designer based on suitable engineering considerations or mission-dependent requirements. $f$ has no dead band (this restriction can be relaxed).

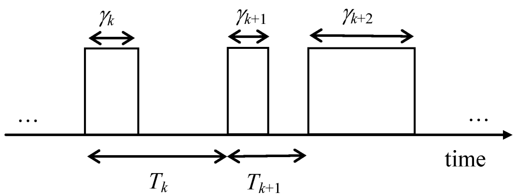


## Pulse modulation and switching constraints

- Some pulse modulators are composed of a Schmitt Trigger, a linear filter and a feedback loop. Others make use of **modulation curves** implementable in software or firmware.
- In-depth discussion of pulse modulators using modulation curves:
  - E. Noges, P.M. Frank, *Pulsfrequenzmodulierte Regelungssysteme*, Oldenbourg, München, 1975.
- Application to the switching restricted case:
  - K.H. Kienitz, J. Bals, Pulse modulation for attitude control with thrusters subject to switching restrictions, *Aerospace Science and Technology*, v. 9, no. 7, pp. 635-640, 2005.



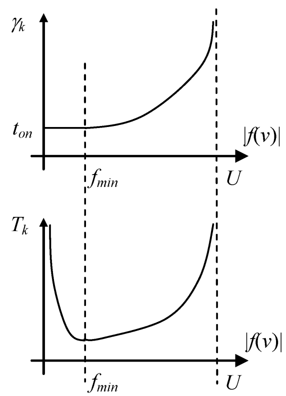
## Pulse modulation and switching constraints




- The proposed pulse modulator scheme is based on the use of two modulation curves, one for $T_k$ and another for $\gamma_k$; both are functions of modulator input $v$.
  - $T_k$: time interval between start of pulse $k$ and start of pulse $k$+1 of $w(t)$;
  - $\gamma_k$: duration of pulse $k$ of $w(t)$.



## Modulation curves




- Modulation curves are defined as functions of $|f(v)|$. The sign of $f(v)$ is chosen as the sign of modulator output $w(t)$:
  - for $f(v) > 0$ pulses of $w(t)$ will be positive
  - for $f(v) < 0$ pulses of $w(t)$ will be negative
- Modulator output zero for $f(v) = 0$.
- Remark: The meaning of $f_{min}$ is clarified later.

## Restrictions and initial considerations

- Restrictions on $T_k$ and $\gamma_k$ result from the problem statement:

$$T_k \geq \gamma_k + t_{off}$$

$$\gamma_k \geq t_{on}$$

- The first inequality guarantees that minimum rest between successive pulses is observed. The second inequality enforces the minimum on-time specification.
- If these inequalities hold, switching restrictions will be satisfied and actuator output will be $u(t) = w(t)U$.



- Hypothesis: constant modulator input $v = v_c$; specification: $u_m = f(v_c)$.
- On the other hand $u_m = \text{sign}[f(v_c)]U\gamma_k/T_k$ . Thus

$$\frac{U\gamma_k}{T_k} = | f(v) |$$

- Fastest possible switching is desirable for good output averaging also in the low frequency range. Thus $T_k = \gamma_k + t_{off}$ shall hold whenever possible. It will not be possible when $\gamma_k = t_{on}$ and additionally

$$| f(v) | < \frac{Ut_{on}}{t_{on} + t_{off}}$$



## Proposal

- The following modulation curves fulfill all aforementioned demands:

$$\gamma_k = \begin{cases} \dfrac{t_{off} \, | f(v) |}{U - | f(v) |} & \text{if } \; v \in \{ v \mid | f(v) | \geq f_{\min} \} \\ t_{on} & \text{otherwise.} \end{cases}$$

$$T_k = \frac{U\gamma_k}{| f(v) |}$$

$$f_{\min} = \frac{Ut_{on}}{t_{on} + t_{off}}$$



## Algorithm

Equations and sign convention for $w(t)$ lead to the following algorithm:

- Modulator initialization at $t = 0$:
  - > Assume $w(t \leq 0) = 0$, evaluate $f[v(0)]$ and calculate $\gamma_k$ and $T_k$.
  - > Initialize $t_{soff}$ (time of last switch off) and $t_{son}$ (time of last switch on) as follows: $t_{son} = - T_k$ and $t_{soff} = - T_k + \gamma_k$.
- After initialization, repeat the following calculation rule (quasi-) continuously:
  - > If $t \geq t_{son} > t_{soff}$ then
    Calculate $\gamma_k$ .
    If $t - t_{son} \leq \gamma_k$ then $w(t) = \text{sign}[f(v)]$, else $w(t) = 0$ and $t_{soff} = t$.
  - >Otherwise
    Calculate $T_k$.
    If $t - t_{soff} \leq T_k - \gamma_k$ then $w(t) = 0$, else $w(t) = sign[f(v)]$ and $t_{son} = t$.

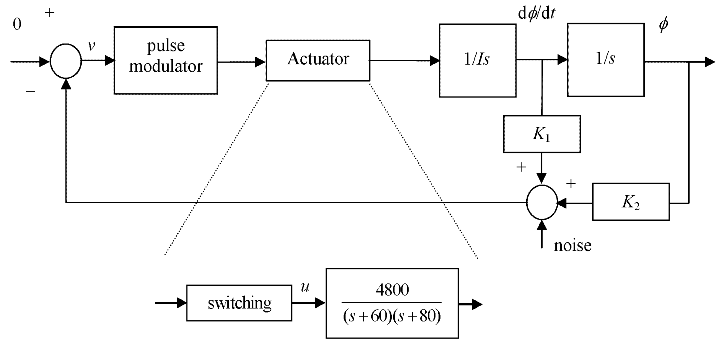
## Flexibility to consider additional specs

- The proposal is easily modified to accommodate additional specifications, such as:
  - maximum switching frequency
  - minimum switching frequency
  - maximum on- time
  - dead band
  - fixed pulse duration
- This approach also works when minimum off time is different between pulses in same and opposite directions.



## Example

Single axis attitude regulation of a rigid body satellite using on-off rocket actuators. $\phi$ designates the attitude angle, $I$ is the inertia moment, $K_1 = 200$ and $K_2 = 10$ are "optimal" controller gains.






- Two integrators model the rigid body dynamics. The block diagram (and simulation) also considers thrust build- up dynamics and thruster switching restrictions.
- Let thrusters produce a torque of 2 [Nm] when switched "on", and be subject to the following switching- time restrictions: minimum on- time $t_{on} = 0.01$ [s]; minimum time- between- pulses of $t_{off} = 0.05$ [s]. Furthermore $I = 1000$ [kg m$^2$].
- Let the goal of pulse modulation in this example be actuator quasi- linearization in order to implement the continuous control law defined by $K_1(\mathrm{d}\phi/\mathrm{d}t) + K_2\phi$.



In order to quasi-linearise the actuator, $f(v)$ was chosen as:

$$f(v) = \begin{cases} 2\,\mathrm{sign}(v) & ,|v| > 2 \\ v & ,2 \geq |v| > 0.04 \\ 0 & ,|v| \leq 0.04 \end{cases}$$

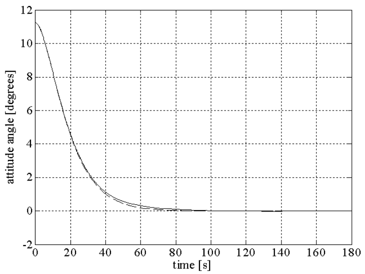
The dead band of 0.04 was adopted to keep actuators switched off for small attitude rates and errors. Such use of dead bands is common procedure and allows for fuel saving.

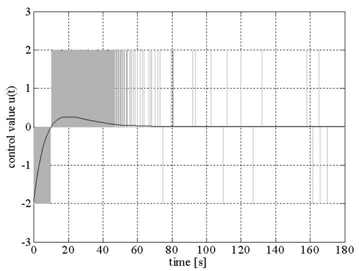
The characteristics of the pulse modulator curves were chosen according to the proposal.

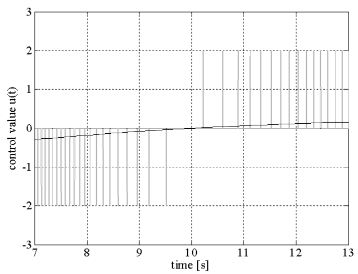
$$\gamma_k = \begin{cases} \dfrac{0.05\,|f(v)|}{2 - |f(v)|} & \text{if } |f(v)| \geq \dfrac{1}{3} \\ 0.01 & \text{otherwise.} \end{cases} \qquad T_k = \frac{2\gamma_k}{|f(v)|}$$

## Results

Responses of the linear double integrator (nominal continuous) system and the full order pulse modulated control system. Initial attitude error: 11.25 deg. All other initial states were zero. Consolidated measurement noise was Gaussian with a variance of $10^{-4}$. Feedback uses measurements in [rd] and [rd/s].






## Comments

- The small differences in response are due to the dead band, the model perturbation $H(s)$ and sensor noise.
- Very large initial conditions may lead to the incapacity of the modulator to produce an output such that $u_m = v$. This will happen for initial conditions that demand $v(0) > 2$. In such cases system transient response will be similar to that of the nominal controlled system with saturated actuator.



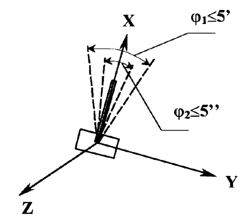
## Pulse modulation - conclusions

- It is possible to determine suitable modulation curves for applications with switching restricted actuators. Switching restrictions such as maximum and/or minimum switching frequencies, maximum and/or minimum on-times and minimum rest between successive pulses can be enforced in a systematic way at design time.
- The proposed approach is intended for quasi-continuous implementation. It uses 2 modulation curves, one for the time interval between successive pulses, one for pulse duration. Arbitrary bounded functions between constant modulator input and averaged modulator output may be specified.

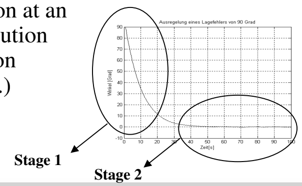


## 3. Direct activation – dynamic behavior patterns

- From: N.N. Antropov, et al., "Pulsed Plasma Thrusters for Spacecraft Attitude and Orbit Control System," *Proceedings of the 26th International Propulsion Conference*, v. 2, pp. 1129-1135, 1999.

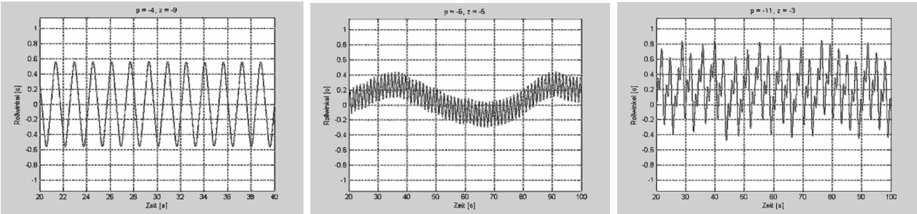



- Attitude control (Stage 1). The spacecraft is turned preliminary to some nominal angle using the thrusters. The accuracy of the axis orientation to the object under study is 2.5' after the first stage. Positioning velocity should be ω > 0.1 [deg/s].
- Attitude stabilization (Stage 2). Attitude stabilization at an accuracy of 2.5". The angular velocity of axis revolution should not exceed $10^{-4}$ [deg/s] during the observation period. (Dead-Band Limit Cycling attitude keeping.)



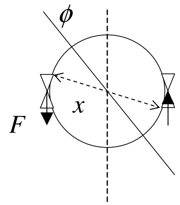
## Patterns

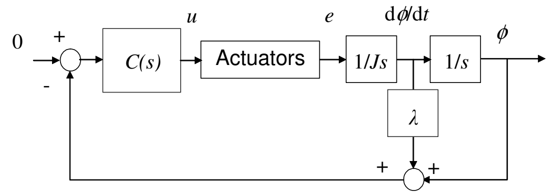
- Instability
- Periodic motion
- Chaotic motion
- „Quasi-periodic like“ motion

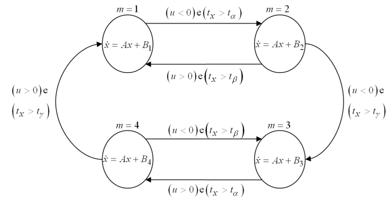





## Example



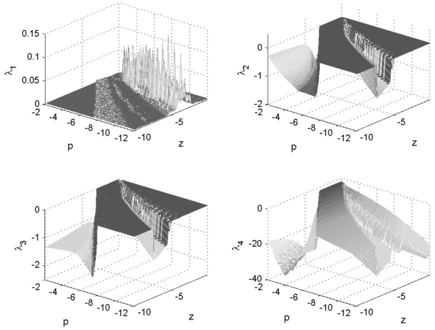

- Choices:
  - $C(s) = (s-z)/(s-p)$
  - $\lambda = 1$
- 1 discrete, 6 continuous states:

$$y(t) = [\phi(t) \quad d\phi(t)/dt \quad e(t) \quad de(t)/dt \quad u(t) \quad t_x(t)]^T$$



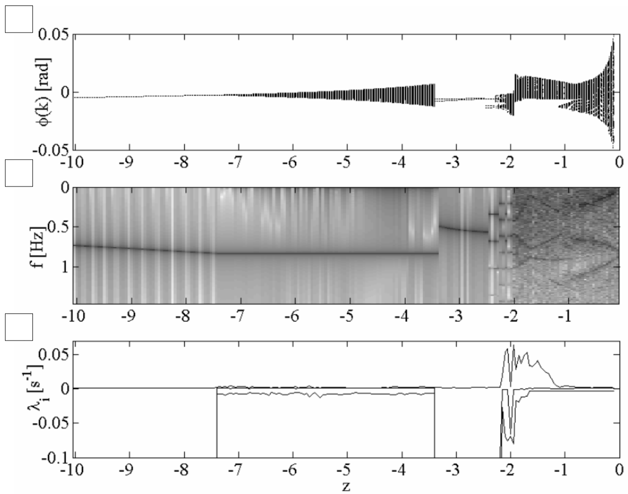
## Lyapunov exponents

- Lyapunov exponents quantitatively and qualitatively characterize dynamical behavior. They are directly related to the convergence / divergence of neighbouring trajectories in phase space. In the linear case the Lyapunov exponents are the real part of the system's eigenvalues.
- The use of Lyapunov exponents is well estalished for smooth systems. Difficulties posed by non-smoothness have been overcome by recent research results and the proposal of adequate calculation methods.
- Biparametric diagrams for the first four Lyapunov exponents → → → → →






## Bifurcations

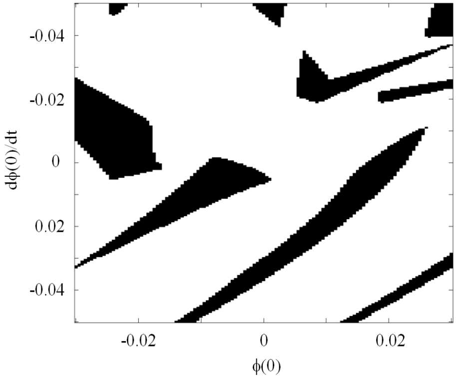



(a) Bifurcation diagram for discretized *f*(k).

(b) The spectral bifurcation diagram of *f*.

(c) The three largest Lyapunov exponents. We fixed *p* = -5.5

## Coexistence of attractors




Transversal section of the basins of attraction for a quasi-periodic-like attractor (white) and for a double-switching periodic attractor (black).

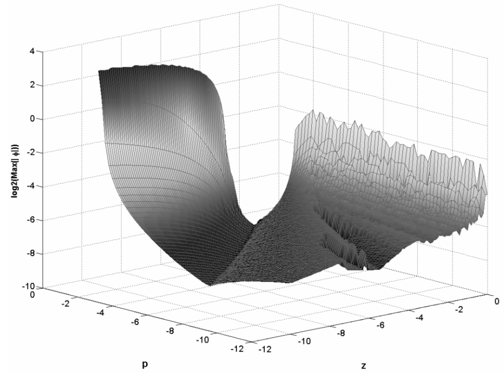
Other initial conditions are zero; at $t = 0$ actuator position is on (+).

p = -5.5; z = -3.3



## Behavior under direct activation - conclusions

- All “complex” motions are disadvantageous from the control point of view because their amplitudes are far away from the minimum one.
- Only single-switching periodic and quasi-periodic-like motions are interesting for control.
- Other motions should be avoided.
- Minimum amplitudes “happen” at the bifurcation frontier periodic motion ↔ quasi-periodic-like motion.



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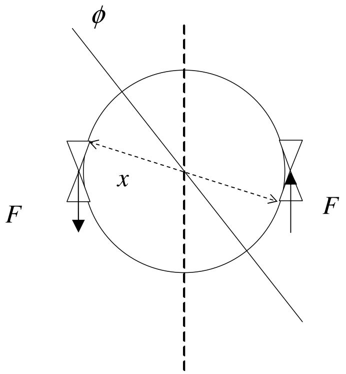
- MESQUITA, A.R.; KIENITZ, K.H.; REMPEL, E.L. - Bifurcation analysis of attitude control systems with switching-constrained actuators, submitted.



## 4. Direct activation – control design considerations

- Defining the setup:
  - Thrust value, i.e. pulse amplitude (**fixed** or not)
  - On-time, i.e. pulse width (fixed or **not**)
  - Thrust build-up dynamics
  - Switching restrictions (i.e. lower limits for pulse width and off-time)
- Dead-band and dead-time are not considered but could also be accomodated

## Example

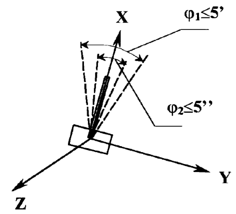



- Switching actuators („relays“) with thrust buil-up dynamics
- Switching restrictions:
  - Minimum pulse duration ($t_{on\_min}$)
  - Minimum time off between actuation using different actuator pairs ($t_{off\_min}$)
  - Minimum time off between actuation using same actuator pair ($t_{off\_min\_s}$)

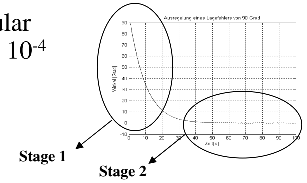


## Typical specification

- Attitude control (Stage 1). The spacecraft is turned preliminary to some nominal angle using the thrusters. The accuracy of the axis orientation to the object under study is 2.5’ after the first stage. Positioning velocity should be ω > 0.1 [deg/s].




- Attitude stabilization (Stage 2). Attitude stabilization at an accuracy of 2.5”. The angular velocity of axis revolution should not exceed $10^{-4}$ [deg/s] during the observation period.



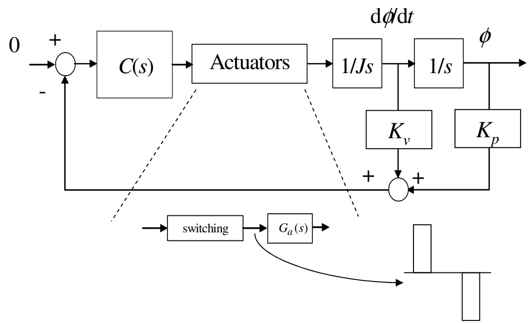



## Summary of specification

- Initial conditions and disturbances shall „die away“ into „well behaved“ persistent motion, i.e. with „small“ amplitudes for attitude error and attitude rate. Maximum allowed values for both variables are given.



## Solution




- **Goal:**
  - Determine suitable values for $K_v$, $K_p$ und $C(s)$.
- **Available tools:**
  - Describing function method, method of Tsypkin/Hamel.

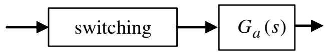
# Using the describing function approach:

Conditions for the limit cycle:

$$1 + G(j\omega)N(A,\omega) = 0$$

$$G(j\omega) = -\frac{1}{N(A,\omega)}$$

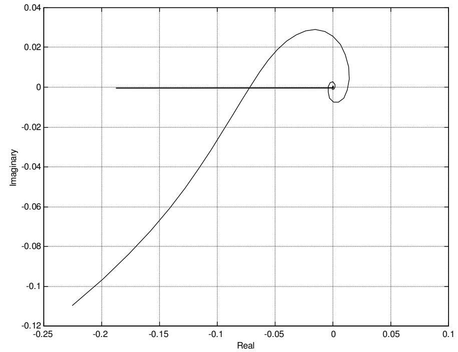
Actuator model:




Describing function:

$$|N(A,\omega)| = \frac{4b}{\pi A}\cos\left(\omega \frac{t_{off\_min}}{2}\right) \qquad \angle N(A,\omega) = -\omega \frac{t_{off\_min}}{2}$$

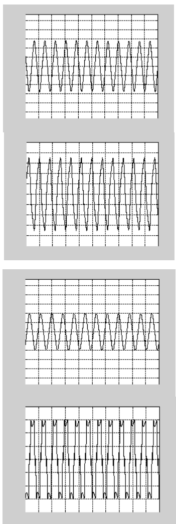





$$|N(A,\omega)| = \frac{4b}{\pi A}\cos\left(\omega \frac{t_{off\_min}}{2}\right) \qquad \angle N(A,\omega) = -\omega \frac{t_{off\_min}}{2}$$

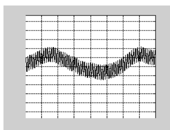


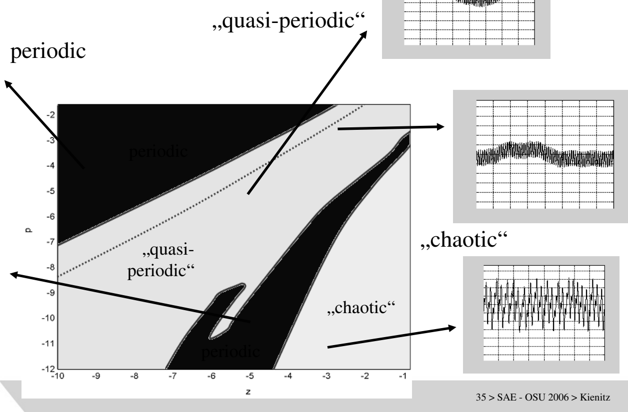
# Bifurcation analysis



Values:

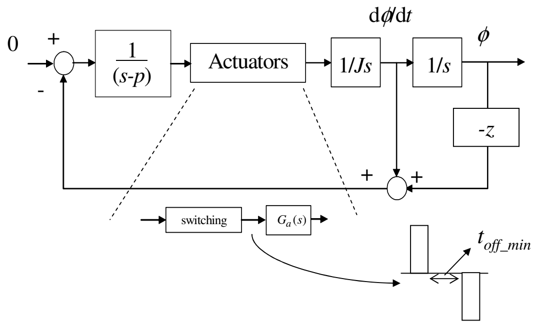
- $C(s) = (s-z)/(s-p)$
- $K_v = K_p = 1$








# Using a Tsypkin/Hamel design approach

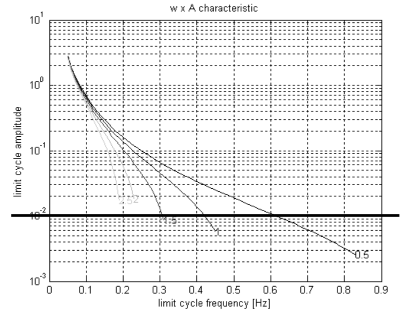



- Goal: determination of „suitable“ values for z, $p$ and $t_{off\_min}$

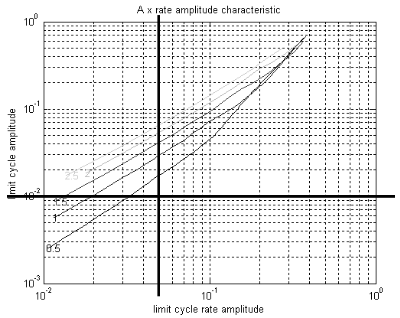
# Facts and specs:

- Limit cycle frequency ⇒ output amplitude ⇒ rate amplitude
- The following diagrams are available (parameterized in $t_{off\_min}$):
  - output amplitude × limit cycle frequency
  - output amplitude × rate amplitude
- These diagrams are controller independent!






- The system can be operated at any one of these points.
- A „suitable“ operation region may be defined using known amplitude specs.



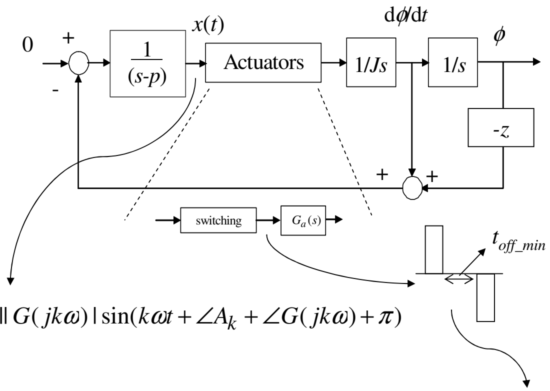

- Low fuel consumption dictates the choice of $t_{off\_min}$ and limit cycle frequency.
- Fuel consumption is proportional to:

  $1\text{-}2.t_{off\_min}/T$,

  with $T > 2[t_{on\_min} + t_{off\_min}]$
- Thus: choose largest possible $t_{off\_min}$, and smallest possible $T$.



# Controller parameter determination

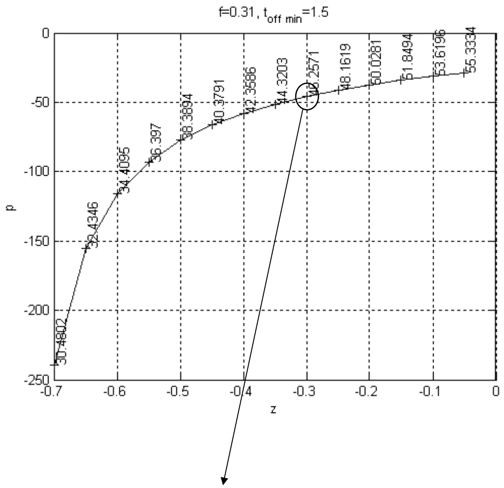



$$x(t) = \sum_{k\ \text{odd}} |A_k||G(jk\omega)|\sin(k\omega t + \angle A_k + \angle G(jk\omega) + \pi)$$

$$\sum_{k\ \text{odd}} |A_k|\sin(k\omega t + \angle A_k)$$

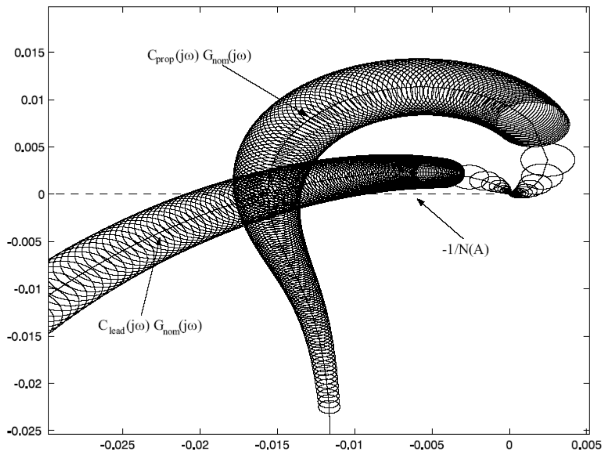
Design equation: $x(-t_{off\_min}) = 0$






- Design choice: $z$ = -0.3; $p$ = - 45.8

## A few words on robust design



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## Current issues, comments

1. Switching constraints can be handled systematically.
2. Switching contraints and „main stream“ hybrid system research.
3. Robustness analysis.
4. Improvement of robust control design procedures:
   - Describing function approach
   - Tsypkin/Hamel method
5. Design of a dead-band (instead of increased $t_{off\_min}$).
6. Application in other areas: electronic switches.