

# **Systems Research in the Aerospace Engineering and Mechanics at the University of Minnesota**

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SAE Aerospace Controls and Guidance Meeting  
28 February 2007

# University of Minnesota

## Aerospace Engineering and Mechanics

### Systems Faculty

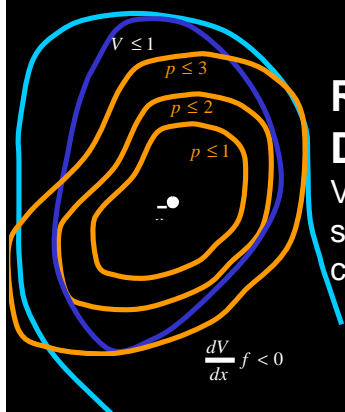
- Gary Balas , Department Head
  - Robust control, real-time embedded systems, flight control
- William Garrard
  - Modeling, flight control, parachutes
- Yiyuan Zhao
  - Optimization, air traffic control, rotorcraft
- Demoz Gebre-Egziabher
  - Navigation, GPS, sensor fusion
- Bernard Mettler
  - Real-time control, planning, rc helicopters and planes

## Current Research

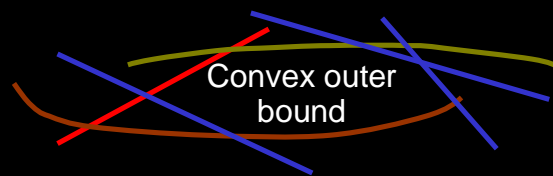
- “Control Reconfiguration and Fault Detection and Isolation Using Linear, Parameter Varying Techniques,” NASA Langley Research Center, NASA Aviation Safety Program, Dr. Christine Belcastro Technical Monitor
- “Stability and Control of Supercavitating Vehicles,” ONR, Dr. Kam Ng Program Manager
- “Development of Analysis Tools for Certification of Flight Control Laws,” joint work with Andy Packard at UC Berkeley and Pete Seiler at Honeywell. This research is being funded by AFOSR.
- Workshop on “Real Time Control of Hybrid Systems: Design, Implementation, Verification, and Validation” in Budapest June 27-28, 2006 sponsored by NSF, Hungarian Academy of Science and Unisaino, Benevento, Italy.

# Development of Analysis Tools for Certification of Flight Control Laws: AFOSR Funding

UC Berkeley, Andrew Packard,  
Honeywell, Pete Seiler; U Minnesota, Gary Balas



**Region-of-attraction**  
**Disturbance-to-error gain**  
Verify set containments in state-space with Sum-of-Squares proof certificates.



**Aid nonconvex proof search (Lyapunov fcn coeffs) with constraints from simulation**

## Long-Term PAYOFF

Direct model-based analysis of nonlinear systems

## OBJECTIVES

- **Develop robustness analysis tools applicable to certification of flight control laws: *quantitative analysis of locally stable, uncertain systems***
- **Complement simulation with Lyapunov-based proof techniques, actively using simulation**
- **Connect Lyapunov-type questions to MilSpec-type measures of robustness and performance**

## APPROACH/TECHNICAL CHALLENGES

- **Analysis based on Lyapunov/storage fcn method**
- **Non-convex sum-of-squares (SOS) optimization**
- **Merge info from conventional simulation-based assessment methods to aid in the nonconvex opt**
- **Unfavorable growth in computation: state order, vector field degree and # of uncertainties.**
- **Reliance on SDP and BMI solvers, which remain under development, unstable and unreliable**

## ACCOMPLISHMENTS/RESULTS

- **Pointwise-max storage functions**
- **Parameter-dependent storage functions**
- **Tangible benefits of employing simulations**

## FUNDING (\$K)

	FY04	FY05	FY06	FY07	FY08
AFOSR Funds	97	141	142		
Other	0	0	0		

## TRANSITIONS

"Stability region analysis using simulations and SOS programming," accepted to 2007 ACC

"Stability region analysis using composite Lyapunov functions and SOS programming," provisionally accepted to *IEEE TAC*

"Local stability analysis using simulations and SOS programming," submitted to *Automatica*

## STUDENTS, POST-DOCS

Ufuk Topcu, Tim Wheeler

## LABORATORY POINT OF CONTACT

Dr. Siva Banda, Dr. David Doman

# Tools for Quantitative, Local Nonlinear Analysis

## Focus over the past 18 months

- Region-of-attraction estimation
- $L_2 \rightarrow L_2$  induced norms
- $L_2 \rightarrow L_\infty$  induced norms

for

- Locally stable, finite-dimensional nonlinear systems, with
  - polynomial vector fields
  - parameter uncertainty (also polynomial)

## Main Tools:

- Lyapunov/HJI formulation
- Sum-of-squares proofs to ensure nonnegativity and set containment
- Semidefinite programming (SDP), Bilinear Matrix Inequalities (BMI)
  - Optimization interface: YALMIP and SOSTOOLS
  - SDP solvers: Sedumi
  - BMIs: using PENBMI

- Constraints provided by simulation

In these slides

## Lots of examples

- 2-d, 3-d and 4-d examples, drawn from literature
- We've demonstrated its performance on a challenging 5-d problem
- Simulation is practical, informative, and aids the search for Lyapunov functions which certify the ROA

## Comparison to Literature

- Only method to incorporate both simulation and certificates of stability
- Generally superior to other general purpose methods

## Moving beyond Region-of-attraction analysis:

Using simulation data to impose necessary conditions on Lyapunov/storage functions that certify

- Local, input/output gain bounds
- Local state reachability
- Attractive invariant sets
- ROA for uncertain systems

extends (conceptually) easily.

# Estimating Region of Attraction

Dynamics, equilibrium point

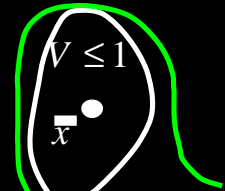
$$\dot{x} = f(x), \quad f(\bar{x}) = 0$$

Find positive-definite  $V$ , with

$$V(\bar{x}) = 1 \text{ is bounded } \left\{ x : \nabla V \cdot f < 0 \right\}$$

nonconvex constraint on  $V$

Then  $\{x : V(x) \leq 1\}$  is invariant, and in the region of attraction of  $\bar{x}$ , denote  $ROA_{\bar{x}}$



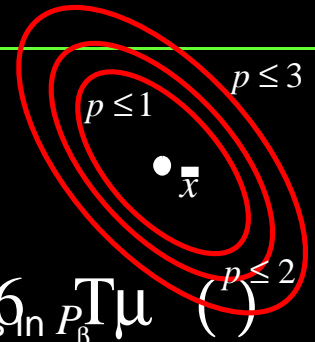
Given a "shape" function  $p$

QUESTION: Fix  $\hat{a} > 0$ , is

$$P_{\beta} := \{x : p(x) \leq \beta\} \subseteq ROA_{\bar{x}}$$

Pragmatic solution:

- run  $N$  sims; starting from samples in  $P_{\beta}$
- If any diverge, then unambiguously "no"
- If all converge, then maybe "yes", perhaps Lyapunov analysis to prove/certify it



How can we use the simulation data to aid in the nonconvex search for a certifying  $V$ ?

Simulations yield

- Collection of convergent trajectories starting in  $P_{\beta}$
- divergent trajectories starting in  $P_{\beta}^c$

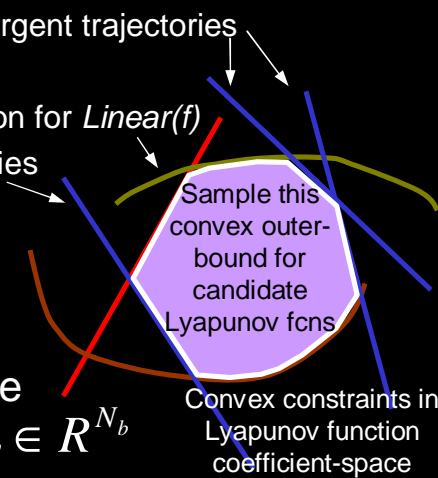
Necessary cond: If  $V$  exists to verify, need

- $V$  1 and decreasing on convergent trajectories
- $V$  0 on all trajectories
- $Quad(V)$  is a Lyapunov function for  $Linear(f)$
- $V$  1 on the divergent trajectories

Linearly parametrize

$$V(x) = \sum_{k=1}^{N_b} \alpha_k \phi_k(x)$$

Necessary conditions are convex constraints on  $\alpha \in R^{N_b}$



Each candidate  $V$  certifies some ROA

$$\beta_{cert, V} := \max \beta \text{ such that } \exists \gamma \text{ satisfying } \{x : p(x) \leq \beta\} \subseteq \{x : V(x) \leq \gamma\} \subseteq \{x : \dot{V}(x) < 0\}$$

Assess in 2 steps, using positivity & sum-of-squares (SOS) optimization to enforce subsets

- SOS optimization ( $s_1, s_2$ ) to maximize the level-set condition on  $V$  (with  $\beta_1, \beta_2 := \beta^{-6} x^T x$ )
- SOS optimization ( $s_3$ ) to maximize condition on  $p, V$

Furthermore, bilinear matrix inequality (BMI) solvers can be initialized with these, and further optimization (adjusting  $V$  too) be performed.

# 5-state aircraft example

Aircraft: Short period longitudinal model, pitch axis, with 2-state dynamic inversion

$$x = \begin{bmatrix} \alpha \\ \eta_1 \\ \eta_2 \\ \theta \end{bmatrix} \quad \dot{x} = Ax + \begin{bmatrix} \text{controller} \\ .82x_1x_2 + .44x_2^2 - .037x_2x_3 - .082x_2x_4 + .015x_2x_5 + .22x_2^3 \\ 0.11x_2x_5 - 0.054x_5^2 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad A = \begin{bmatrix} -3 & -1.3 & 1.3 & 3 & -.6 \\ .91 & -.64 & .04 & .089 & -.017 \\ -.060 & 0 & -.060 & .089 & -.017 \\ -.75 & 0 & 0 & 0 & -.28 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$p(x) := x^T x$$

Simple form for shape factor:

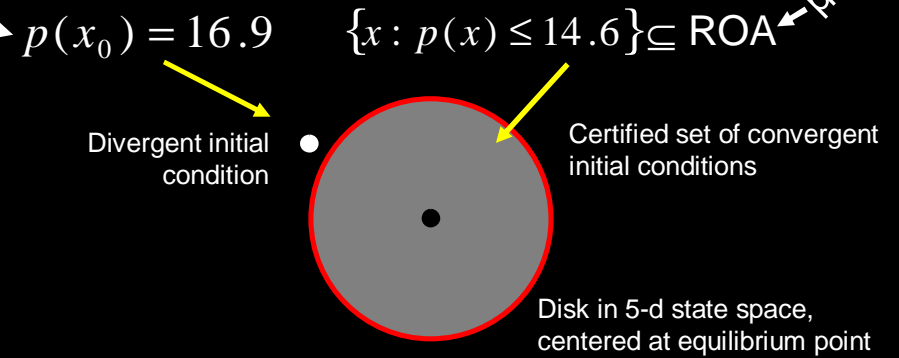
Different Lyapunov function structures

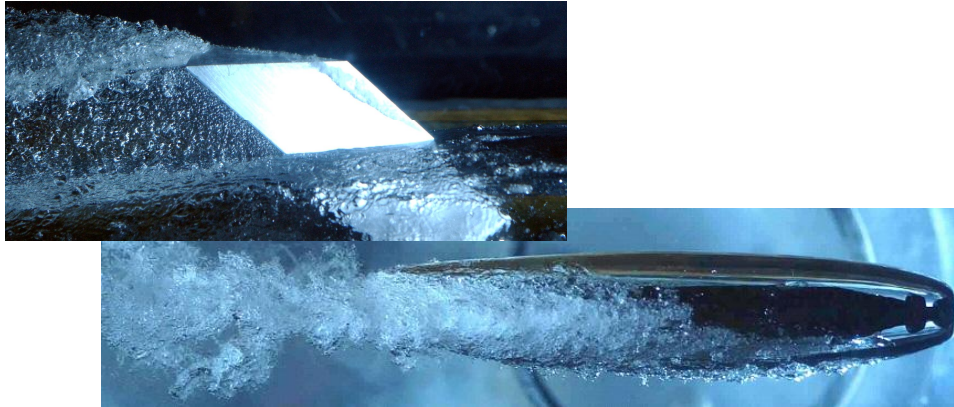
- Quadratic ( $\hat{a}_{cert}=8.6$ )
- pointwise-max quadratics ( $\hat{a}_{cert}=8.6$ )
- Quadratic+Quartic ( $\hat{a}_{cert}=12.2$ )
- Fully quartic (quadratic + cubic + quartic)
  - $\hat{a}_{cert}=14.6$

Other approaches have deficiencies

- Directly use commercial BMI solver (PENBMI)
  - $\hat{a}_{cert}=15.2$ , but...
  - 38 hours!!!
- Iteration from "random" starting point
  - Use  $P$  from  $A^T P + PA = -I$
  - Initialize  $V(x) = x^T P x + 0.001 \sum_{i=1}^5 x_i^4$
  - 30 iterations,  $\hat{a}_{cert}=8.6$

4000 simulations	5 minutes
Form LP/ConvexP	3 minutes
Get a feasible point	5 minutes
Assess answer with V	2 minutes
Iterate from V	3 minutes/iteration, 6 iters
<b>TOTAL</b>	<b>33 minutes</b>





*A research program that integrates cavity physics and control of a supercavitating vehicle on a small, flexible laboratory test bed*

## Objectives

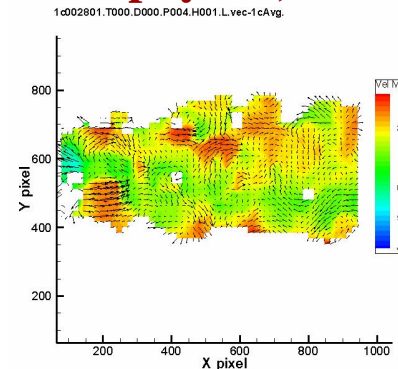
- Integration of super-cavitating dynamics with advanced control strategies for cavity running bodies.
- Explore ventilation physics for startup and target acquisition including computational tools for simulating compressible ventilated flows.
- Real time validation and verification of mathematical models and control strategies.

## Methodology

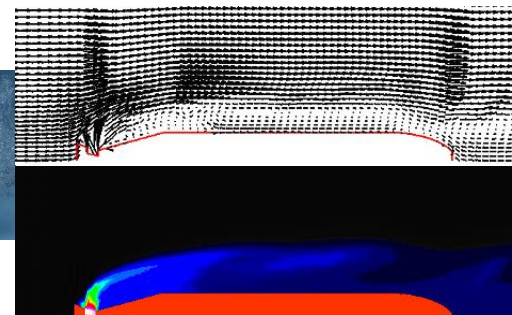
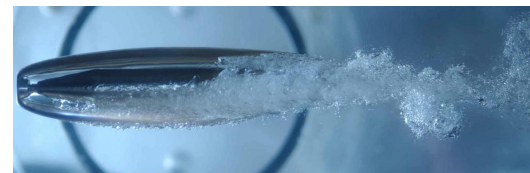
- Nonlinear, robust control design and analysis
- Water tunnel experiments
- Numerical modeling

**The UMN 2 DOF/delay-dependent control model allows investigation of aggressive trajectories limited by: planing requirements, actuator bandwidth, deflection and rate constraints, and performance/robustness of controller. New control theory has been developed to directly address nonlinear interaction between the cavity and body.**

**Water tunnel experimentation includes ventilation physics, characterization of the wake with DPIV**



**A fully compressible LES model provides the necessary computational tools**



**All three elements are integrated**

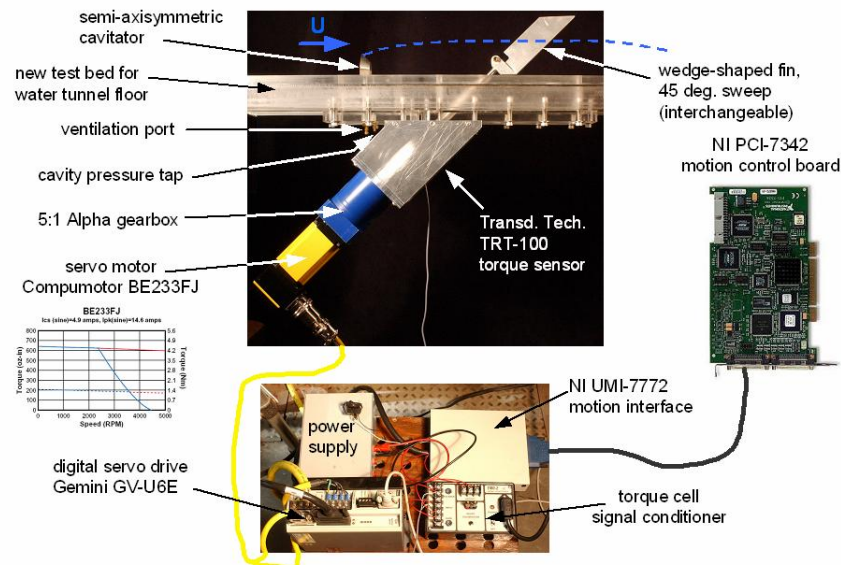
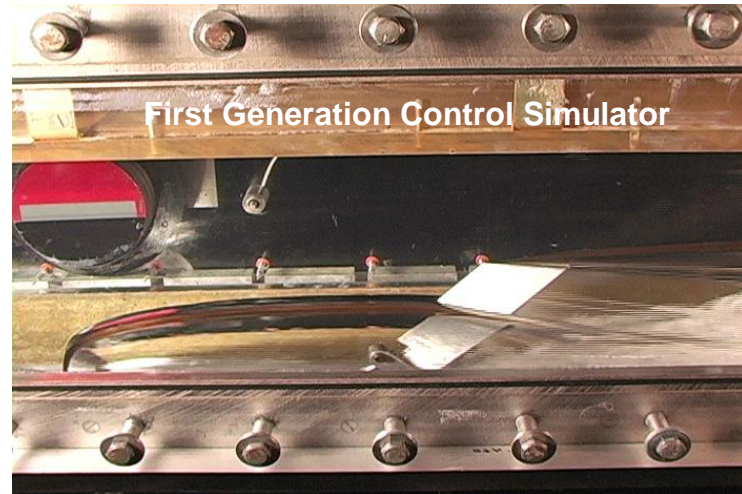


# U of M Control Surface-Cavity Interaction Simulator

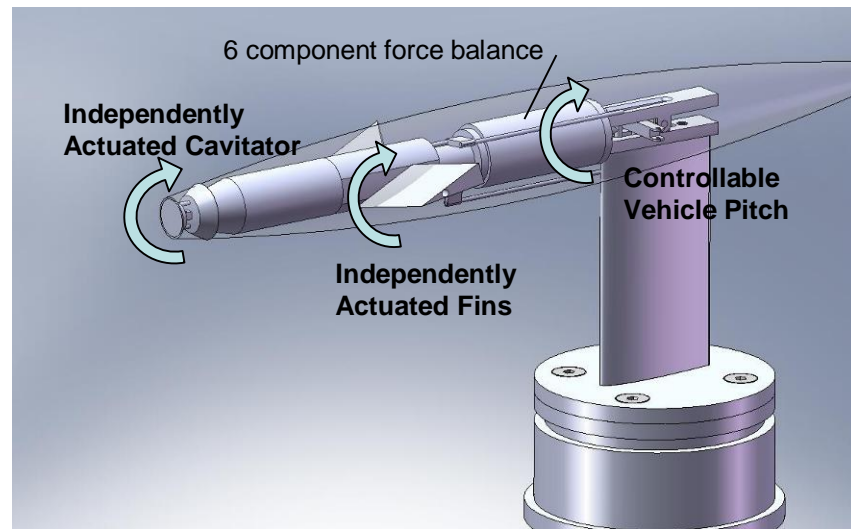
## Integration of the control model with water tunnel experiments:

- allows exploration of tradeoffs between cavitator and fin control
- permits the investigation of the tight coupling between the supercavitating vehicle, cavity impingement and ventilation physics

First generation experiments were made with a window-mounted simulator (to the right and below): a new second generation design is a pitch axis simulator with fully articulated fins and cavitator (bottom right).



First Generation Control Simulator



Second Generation Control Simulator



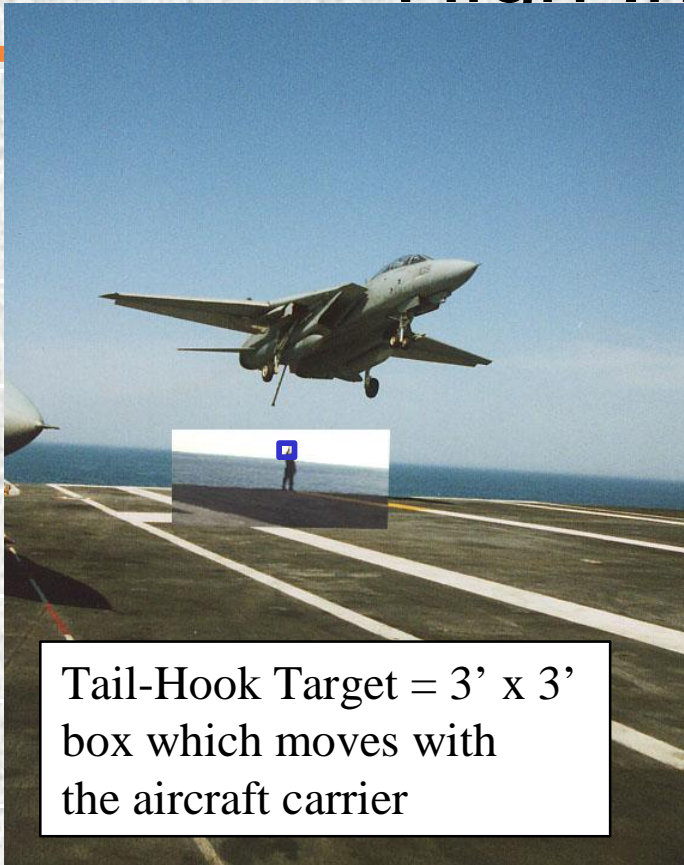
# Navigation and Guidance Research



Demoz Gebre-Egziabher  
Department of Aerospace Engineering and  
Mechanics  
University of Minnesota, Twin Cities

UNIVERSITY OF MINNESOTA

# High Integrity Navigation



- Design and validate high integrity navigation systems for the DoDs Joint Precision Approach and Landing System (JPALS)
  - Precise over bounds on navigation errors.
  - Fault detection and isolation algorithms
  - Methods for fusing information from multiple navigation sensors (GPS, INS, baro-altimeter)
- Sponsor: Lockheed Martin

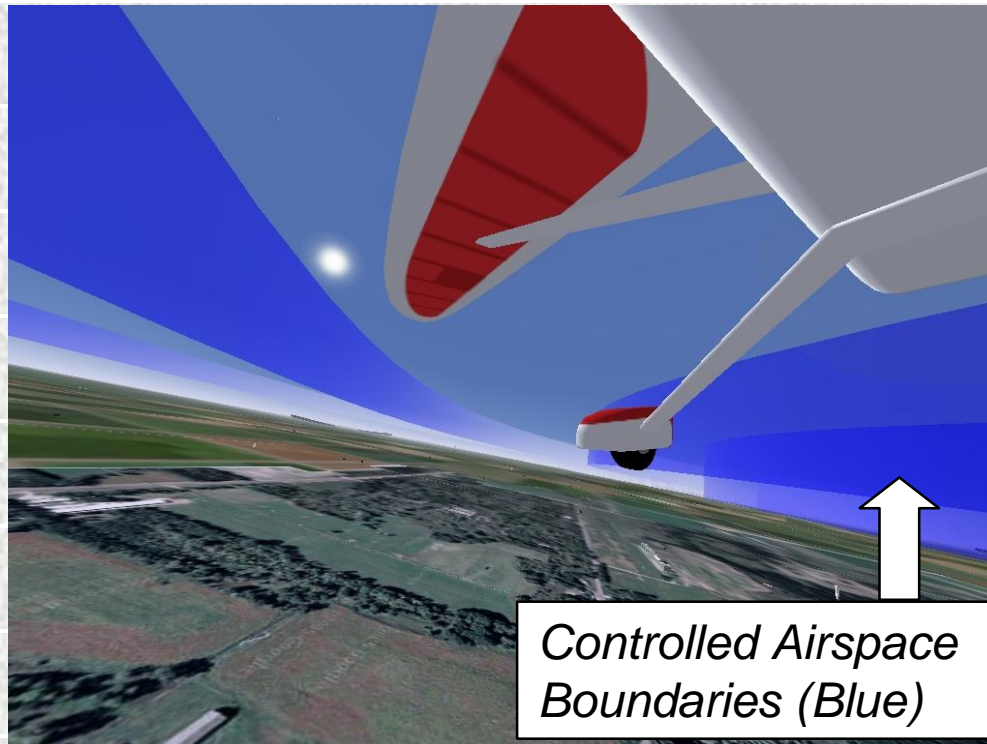
## JPALS Performance Specifications:

Accuracy = 1.1 m Vertical Alarm Limit

Integrity =  $10^{-7}$  Probability Hazardously Misleading Information

Time to Alarm = 1 sec.

# UAV/RPV as ITS Sensor Platforms



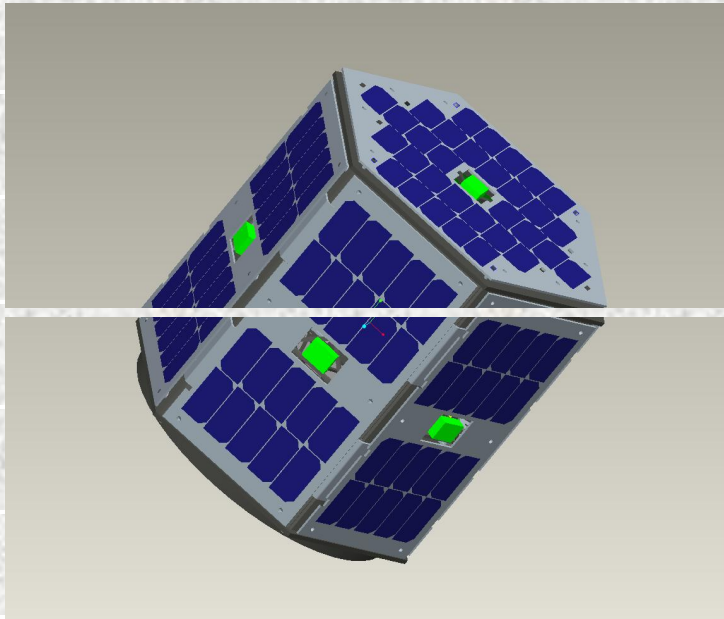
*Synthetic Vision Display developed at U of M as part of this research for remote operation situational awareness of small aerial vehicles. Display fuses satellite imagery and a GIS data base with an open source flight simulator*

- Explore capabilities enabled by Uninhabited Aerial Vehicles (UAV) or Remotely Piloted Vehicles (RPV) in Intelligent Transportation Systems (ITS) applications
  - Data gathering, surveillance.
- Develop “turn-key” sensors and systems which enable use of them in ITS applications
- Explore regulatory and technical issues associated with safely operating RPV/UAV close to the National Airspace System (NAS)

# Micro- and Nano-Satellite Design

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## Minnesat



■ GPS Antenna

■ Solar Cell

- Design of systems and algorithms for ultra-short baseline GPS attitude determination systems for micro- and nano-satellites.
  - Baselines on the order of one-wavelength.
  - Non-aligned antenna arrays.
  - Modification of COTS components
- Sponsor: US Air Force Research Labs, Space Vehicle Directorate (University Nanosat Program)

# *Aerospace Engineering & Mechanics*

**William L. Garrard**

**Dynamics and Control of Aerospace  
Vehicles**

**Flight Mechanics of Parachute Systems**



# *Aerospace Engineering & Mechanics*

## Prediction of Parachute Opening Dynamics



# *Aerospace Engineering & Mechanics*

## Azimuth Pointing Control Of A Balloon-Borne Stabilized Platform



# *Guidance and Control of Autonomous Vehicles*

**October 6, 2006**



**Bernard Mettler**

**Assistant Professor**

**Department of Aerospace Engineering and Mechanics**

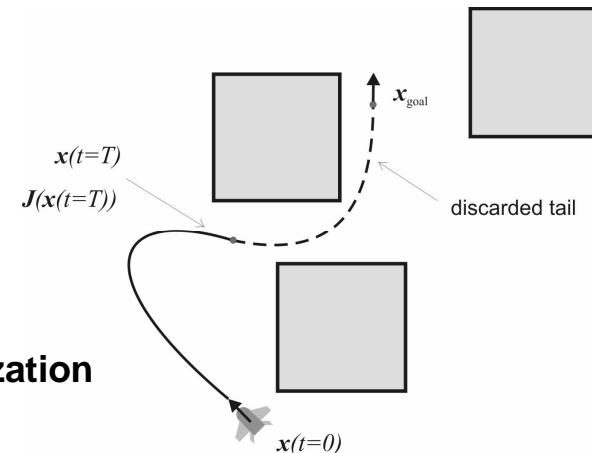
**University of Minnesota**

## Requirements for Autonomous Vehicle Operation

- **Mobility and agility are fundamental to autonomous vehicles**
  - negotiating complex terrain environments
  - handle difficult operational conditions (i.e. atmospheric disturbances)
  - making full use of vehicle dynamic capabilities
- **Involves interplay between lower-level flight control and higher-level trajectory planning**
- **Requires technique that integrate local and global scales**

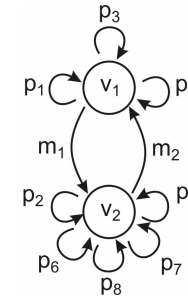
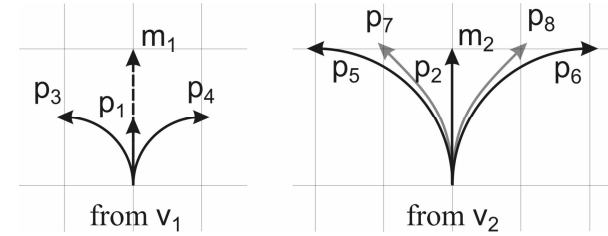
## Receding Horizon Trajectory Optimization with a Cost-to-go Function

- **Principle**
  - perform trajectory optimization over finite horizon  $T$
  - capture discarded trajectory tail with a *cost-to-go* (CTG)  $J(x(t+T))$
- **Provides a rigorous framework to combine on- and offline optimization**
  - Offline (near real time  $O(\text{secs})$ ): computation of CTG
  - Online: computation of the control action
- **Goal: Approach the performance of infinite horizon optimization with less computational burden**
- **Key question: how to compute the CTG function ?**



# Cost-to-go Computation

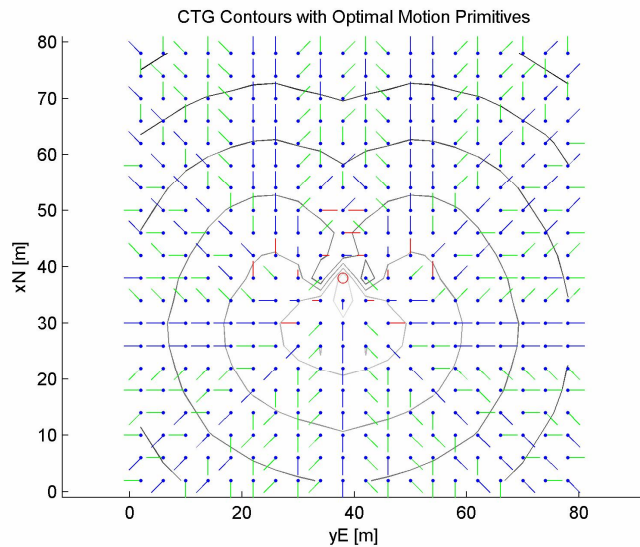
- **Finite-state model to capture vehicle maneuvering capabilities. Example:**
  - Quantized speeds:  $v_1, v_2, \dots$
  - Capabilities at each speed represented by feasible turning and linear accelerations
  - Discrete headings ( $\delta/4$  resolution)
- **Compute cost-to-go function with Dynamic Programming**



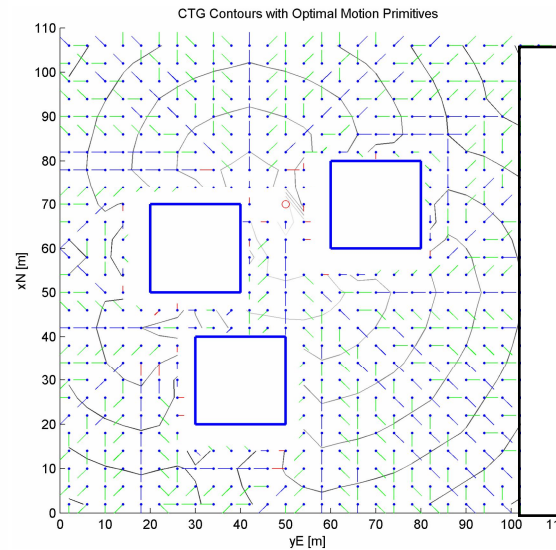
## Examples cost-to-go Maps

goal: circle "o" heading North

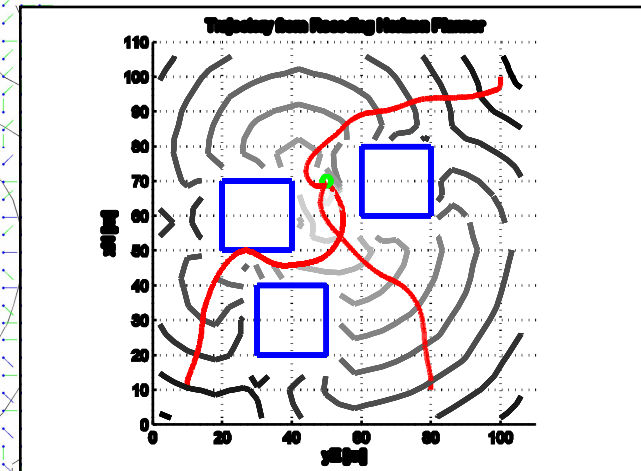
Open space CTG



Space with Obstacles CTG



Trajectories based on CTG



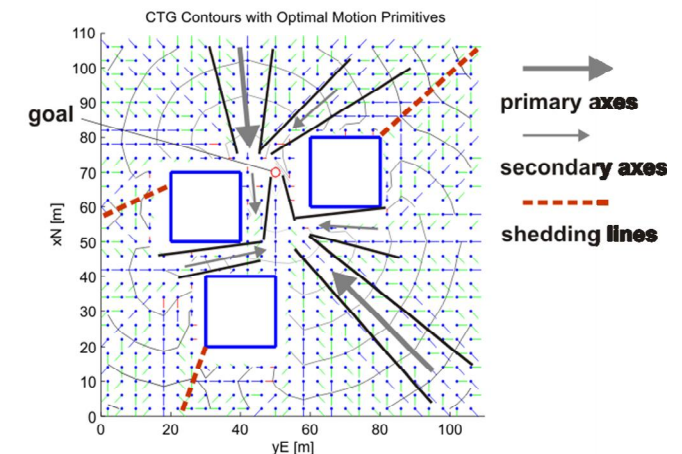
## Key Features

- **Computationally efficient:**
  - CTG computed  $O(\text{secs})$
- **No special environment model required (works with digital elevation map)**
- **Captures global features of planning problem from standpoint of vehicle capabilities**
  - Environment is “resolved” by characteristic vehicle motions
  - Provides a way to partition the environment (virtual roadmaps)
- **Accurate CTG map can be used directly with a control policy**
- **Provides a framework to study interplay between spatial constraints and dynamic behavior**

## Ongoing Work

1. **Find adequate level of approximations in CTG computation, i.e., given vehicle capabilities and operational requirements, determine:**
  - Resolution of cost-to-go map, number/type of motion primitives
  - Length of the optimization horizon and update interval
2. **Exploit global CTG structures**
  - Partitions and “virtual roadmaps” are based on vector field characteristics
  - Enable high-level decision making (tactical)
3. **Develop efficient algorithms for online optimization**
4. **Robustness to stochastic effects**
  - Uncertainty about environment knowledge
  - Disturbances

### CTG Partitions/Structures



## Indoor Flight Experiment Facility

- **Vision based tracking system used for aircraft positioning and attitude**
  - No on-board instrumentation required
  - Makes possible to use micro helicopters and airplanes
- **Create controlled experimental conditions**
  - Scenarios with real or virtual 3D environments
  - Jets to simulate wind disturbances
  - Interactive human-autonomous experiments

### Other Aircraft under Investigation

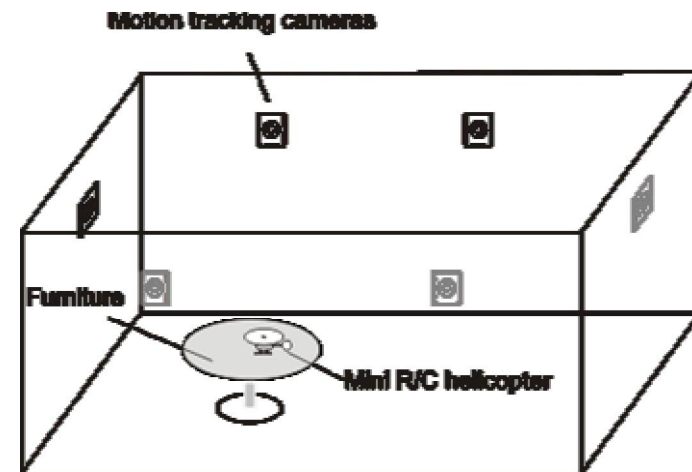
Micron FP Helicopter (50g)



Plantraco Butterfly (3g)

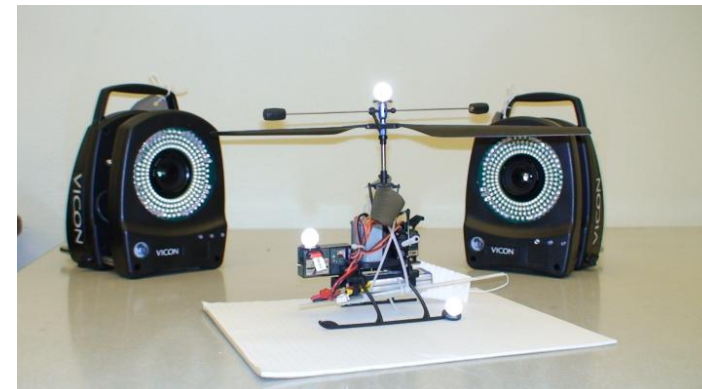


### Indoor flight lab setup



### Test Helicopter with cameras

E-Flight Blade CX (200g) with Vicon's MX 40 cameras



# Current Research Activities of Yiyuan J. Zhao University of Minnesota

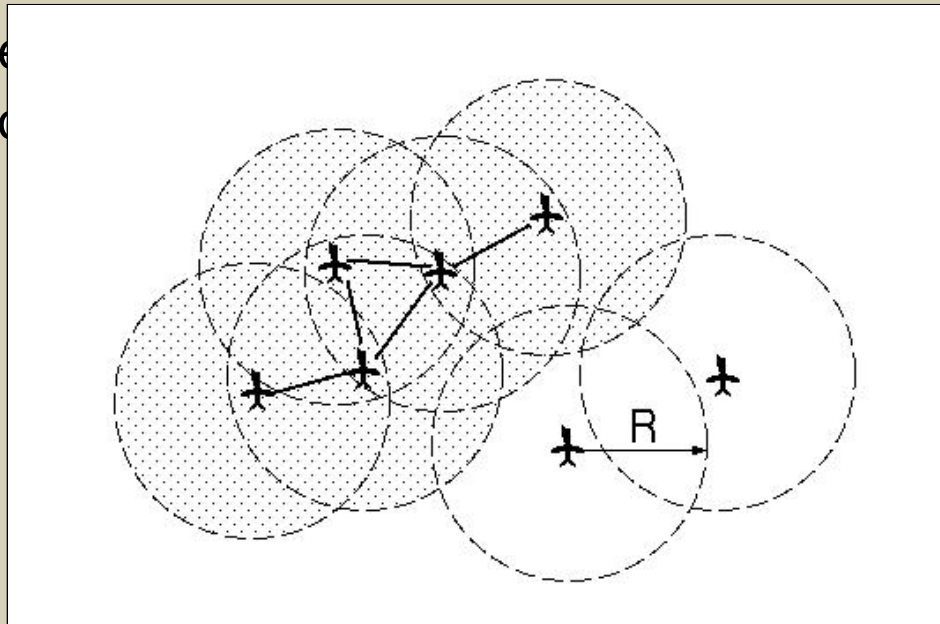


- ∅ Distributed Dynamic Optimization Across Airborne Networks
- ∅ Optimal UAV Flights in Wind

# Distributed Dynamic Optimization Across Airborne Networks

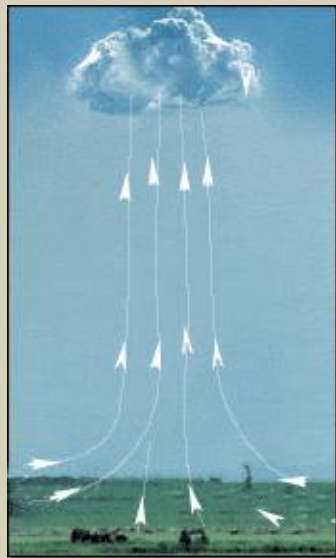
- ∅ Airborne Networks: Ad Hoc Wireless Networks Among Aircraft
- ∅ Each Vehicle Calculates Its Own Optimal Flight Trajectory

- ┆ To optimize
  - ┆ To avoid collision
- main team task

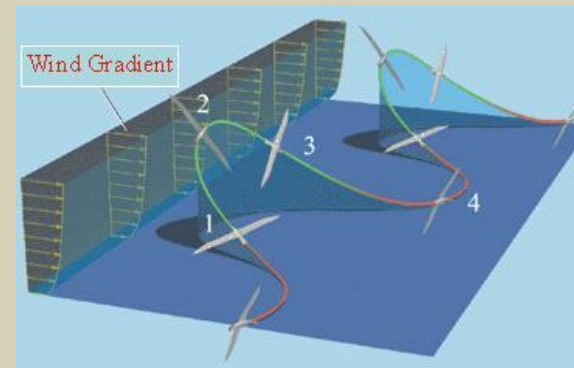


# Optimal Unmanned Aerial Vehicle Flight in Wind

- ∅ Potential Benefits of Wind Energy Utilization in UAV Flight
  - | Reduced fuel consumption/prolonged flight/increased range



Thermals



Wind Gradients



Mountain Waves