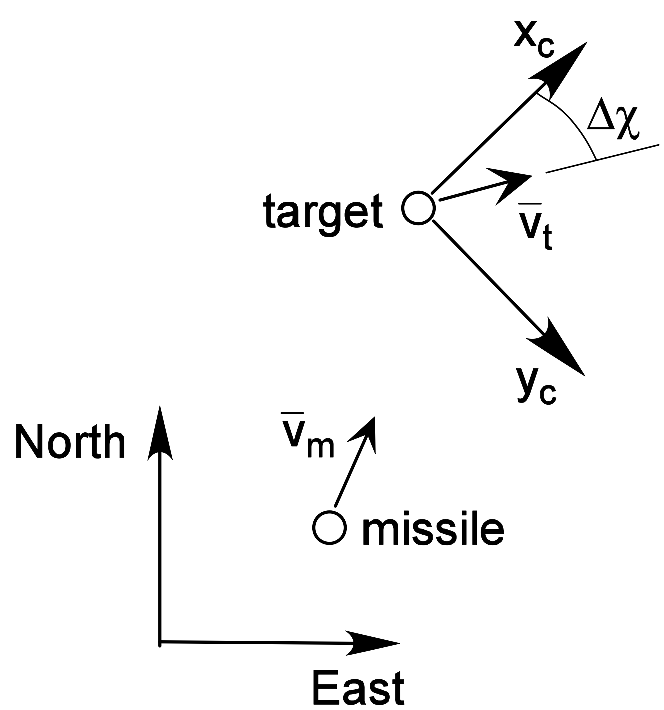
Integrated Guidance-Control of Missiles

Research Supported by NSWC-Dahlgren and MDA
Technical Monitor: Ernest J. Ohlmeyer, NSWCDD

By
P. K. Menon, V. S. S. Vaddi, G. D. Sweriduk
Optimal Synthesis Inc.
868 San Antonio Road
Palo Alto, CA 94303-4622
(650) 213-8585, ext. 201
www.optisyn.com
menon@optisyn.com

Objective of Missile Guidance & Control

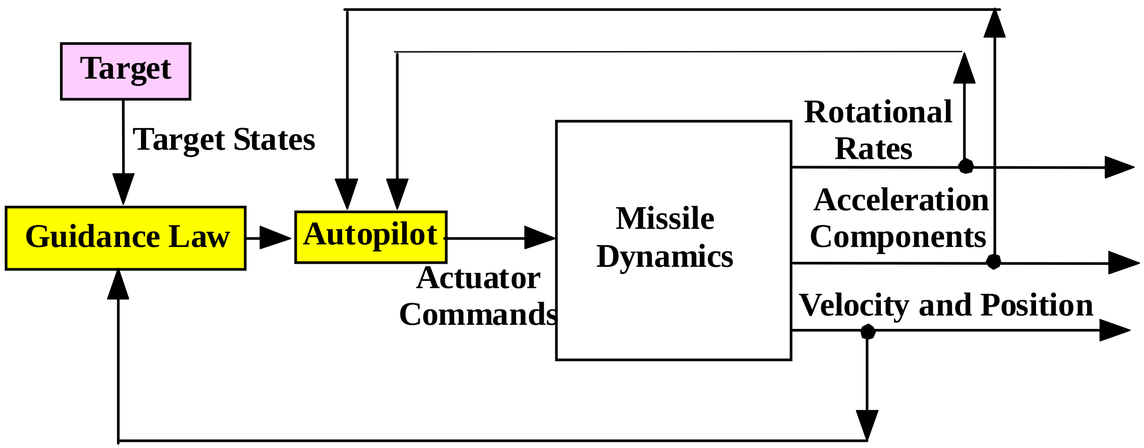


Drive the Target Referenced Missile Position Components to Zero in Specified Time/Range

Outline

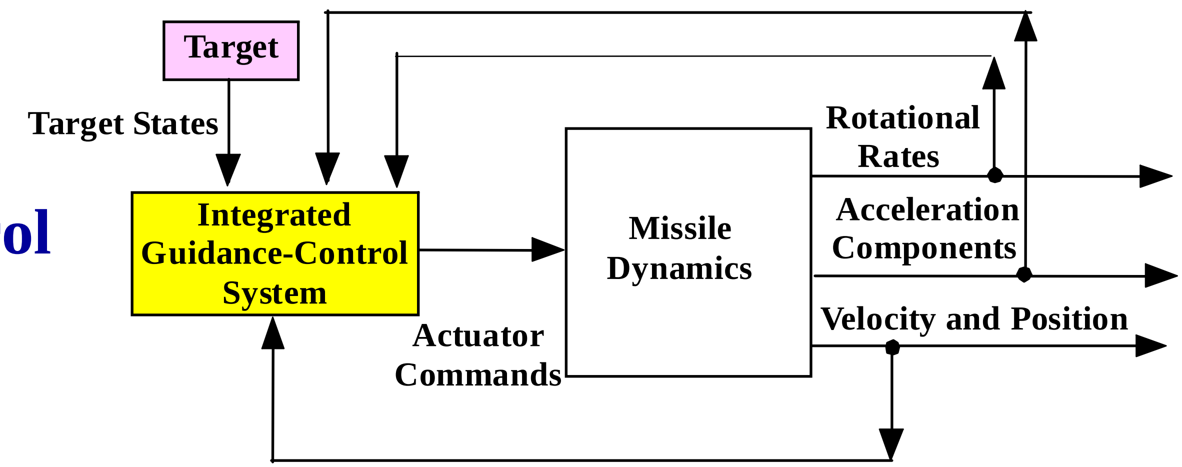
- Integrated Guidance-Control (IGC) Systems
- Benefits and Difficulties
- IGC Design Methods
  - Linear and Nonlinear Techniques
- IGC Design Examples
  - Air-to-Air Missile
  - Internally Actuated Kinetic Warhead
- Summary and Conclusions

Conventional Vs Integrated Guidance-Control Systems



Conventional Guidance and Control System

Integrated Guidance-Control System



Conventional Guidance & Control System

- **Partial Feedback of the Missile States to Guidance System, No Direct Feedback of the Target States to the Autopilot:**
  - **Autopilot Time Constant has a Significant Influence on the Miss Distance.**

- **Potential for Instabilities in the G&C System Induced by the Integral Guidance Command Tracking Error Feedback in the Presence of Actuator Saturation.**

- **Iterative G&C System Design (Finite-Interval Guidance Law, Infinite Horizon Autopilot)**

Benefits of Integrated Guidance-Control Systems

- **Simplifies the G&C Design Process**
- **Can Takes Advantage of the Synergism Between Guidance and Autopilot Functions.**
  - **Adjusts the Autopilot Response to Accommodate Guidance Demands,**
  - **Avoids Autopilot Command Saturation Due to Agile Targets Maneuvers**
  - **Provides Better Performance Margins.**
- **Can Meet Enhanced Performance Requirements: E.G., Execute Terminal Maneuvers to Control Final Orientation of the Missile Velocity Vector W.R.T the Target.**
- **Requires Substantial Onboard Computational Capability.**

Difficulties in the Design of Integrated Guidance-Control Systems

- **High-Order Nonlinear Dynamics**
- **Finite-Interval Control Problem**

IGC Design using Taylor Series Linearized Dynamics(1992)

<u>Previous Work by the Authors:</u>

1. SDRE-Based IGC Formulation (1997, 1999)

2. Infinite-Horizon IGC Formulations & Design with Feedback Linearized Dynamics (1998 - 2002).

3. Finite-Horizon IGC Formulation (2003)

4. Multi-Stepping Algorithm for Finite-Horizon IGC (2004)

<u>Most Recent Research (2006):</u>

Finite-Horizon *Robust* IGC Formulation and Solution Using Feedback Linearization

Integrated Guidance-Control System Design Methods

- **Linear Design Techniques Require Gain Scheduling with respect to Dynamic and Kinematic States**

- **Nonlinear Design Methods:**

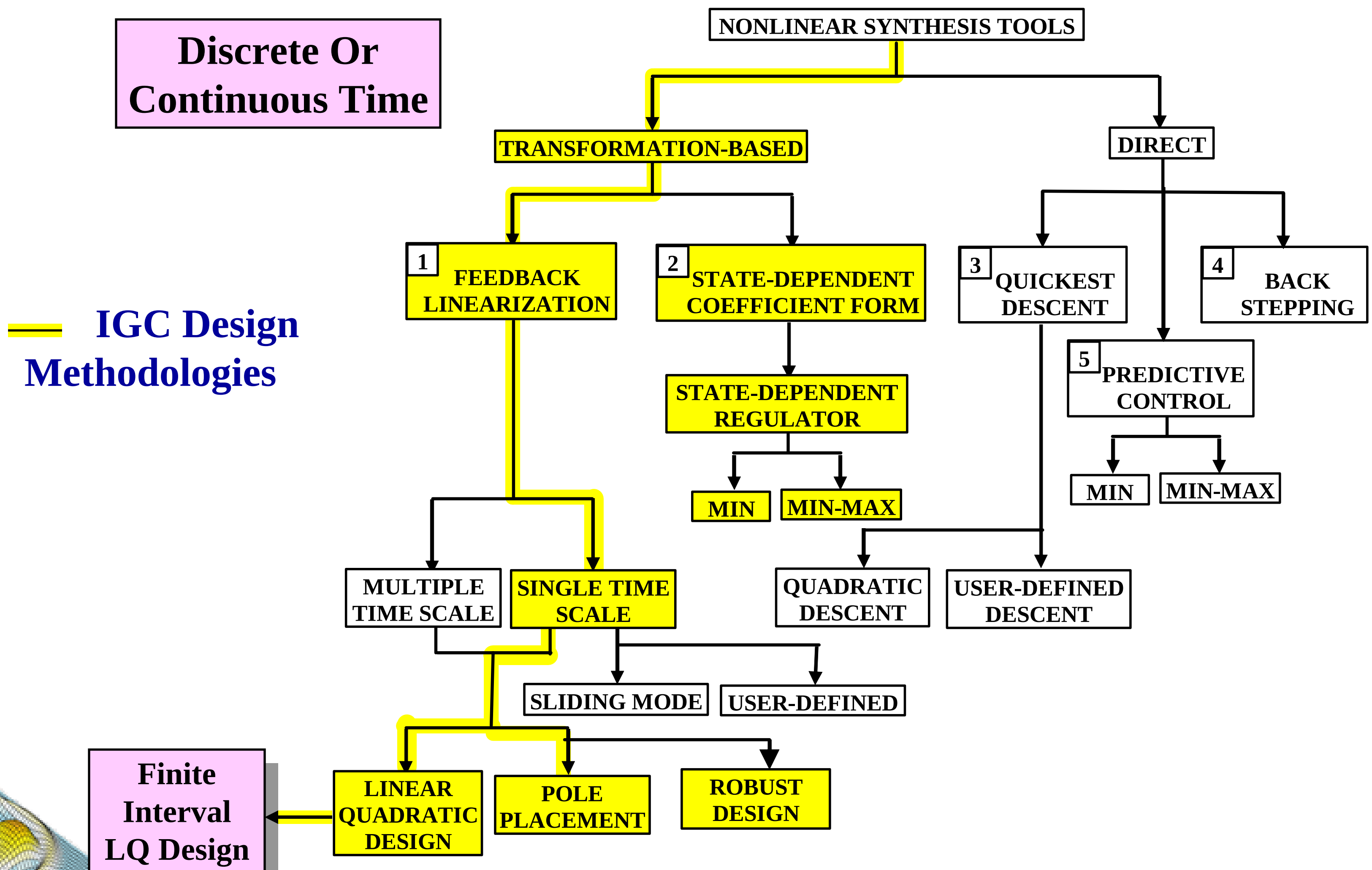
  - **Proven Methods are Not Available.**
  - **Tedious and Extensive Algebraic Manipulations may be Required.**
  - **Complex to Design and Analyze.**
  - **Computational Tools are Not Available.**

- **Recently Developed Numerical Methods and Design Software for Nonlinear Control Techniques can Ameliorate these Difficulties.**

- **This Software (*Nonlinear Synthesis Tools*) is used in IGC System Research.**

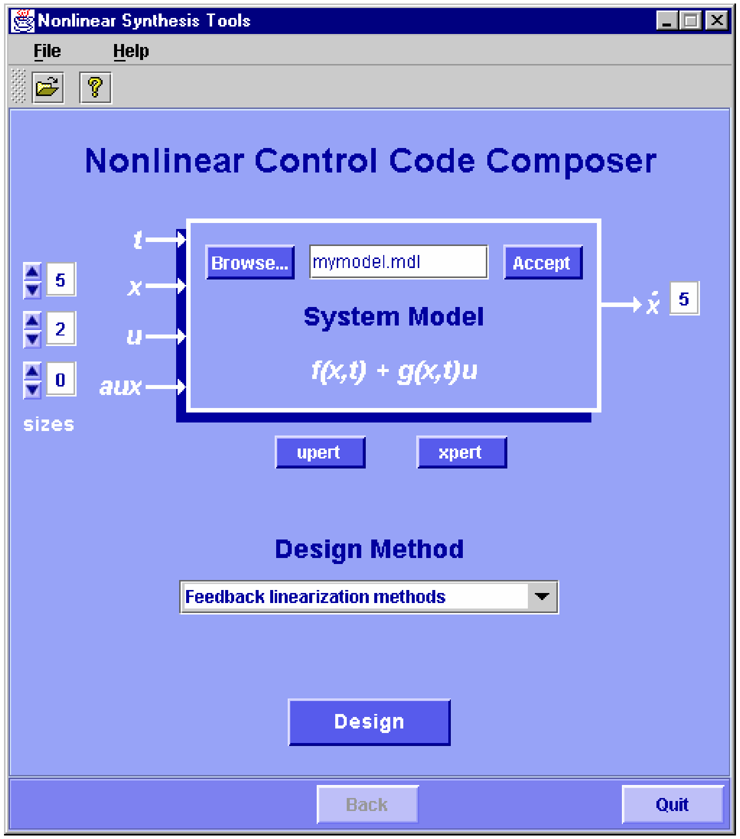
- **Can Automatically Generate Error-Free C Code for Real-Time Implementation of Nonlinear Control Systems**

IGC System Design Using *Nonlinear Synthesis Tools™*

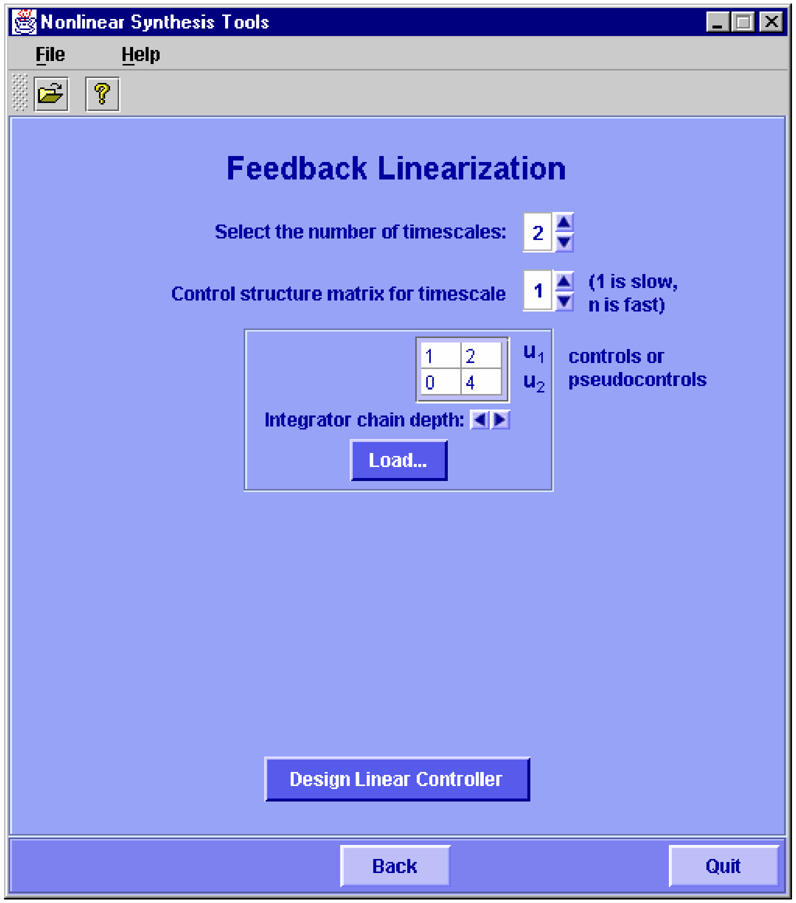


Optimal Synthesis Inc.

Graphical User Interface for Nonlinear Control Design



Main GUI Panel

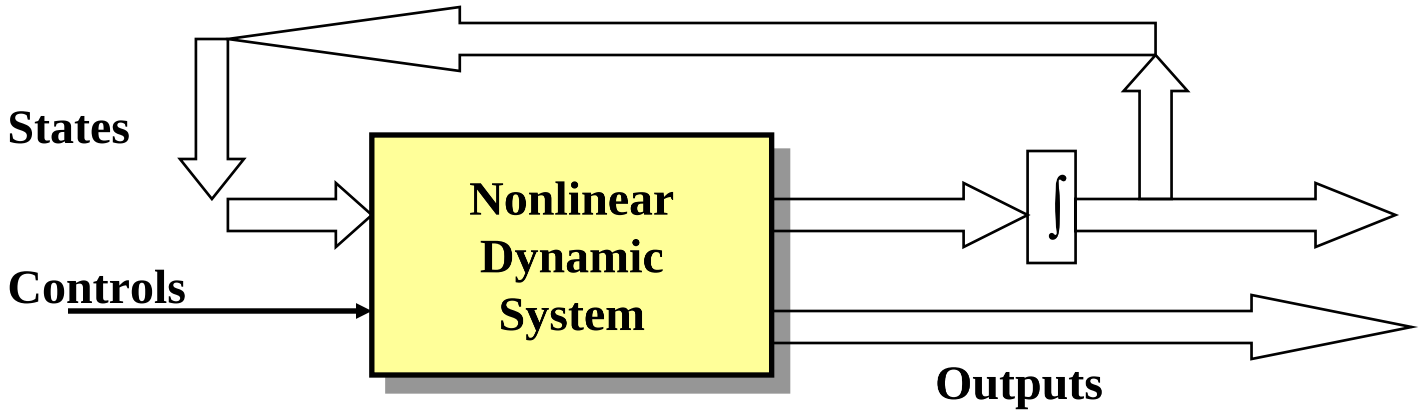


GUI Panel for Feedback Linearization Method

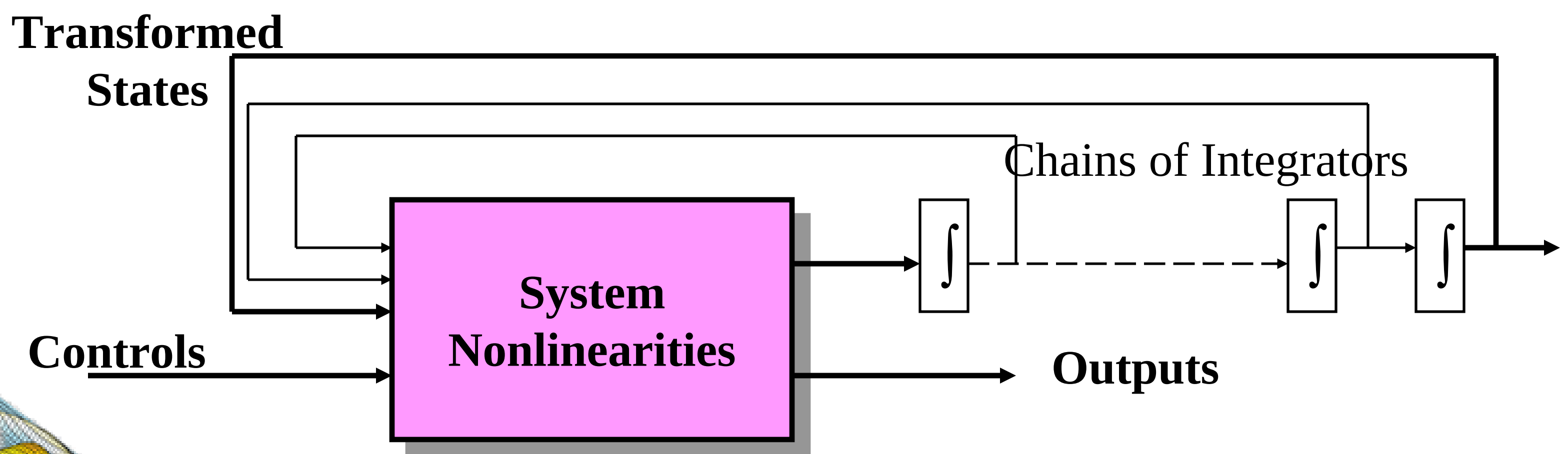
Optimal Synthesis Inc.

Feedback Linearization of Dynamics

- **Given the Nonlinear Dynamic System:** $\dot{x} = f(x) + g(x)\,u$



- **Transform the Model to Brunovsky Canonical Form**



SDRE Design Technique

- **Given Nonlinear Dynamic System:** $\dot{x} = f(x) + g(x)\,u$
- **Transform the System Model to the *SDC* Form:**

$$\dot{x} = A(x)\,x + g(x)\,u$$

- **Solve *State Dependent Riccati* Equation:**

$$A^T(x)\,P + PA(x) - P\,g(x)\,R^{-1}(x)\,g^T(x)\,P + Q(x) = 0$$

Corresponding to the Performance Index

$$\int_0^\infty [x^T Q(x)\,x + u^T R(x)\,u]\,dt$$

- **Compute State Dependent Control Law As:**

$$u = R^{-1}(x)\,g(x)P(x)\,x$$

- **Select *Q(x)* and *R(x)* Matrices to Satisfy:**

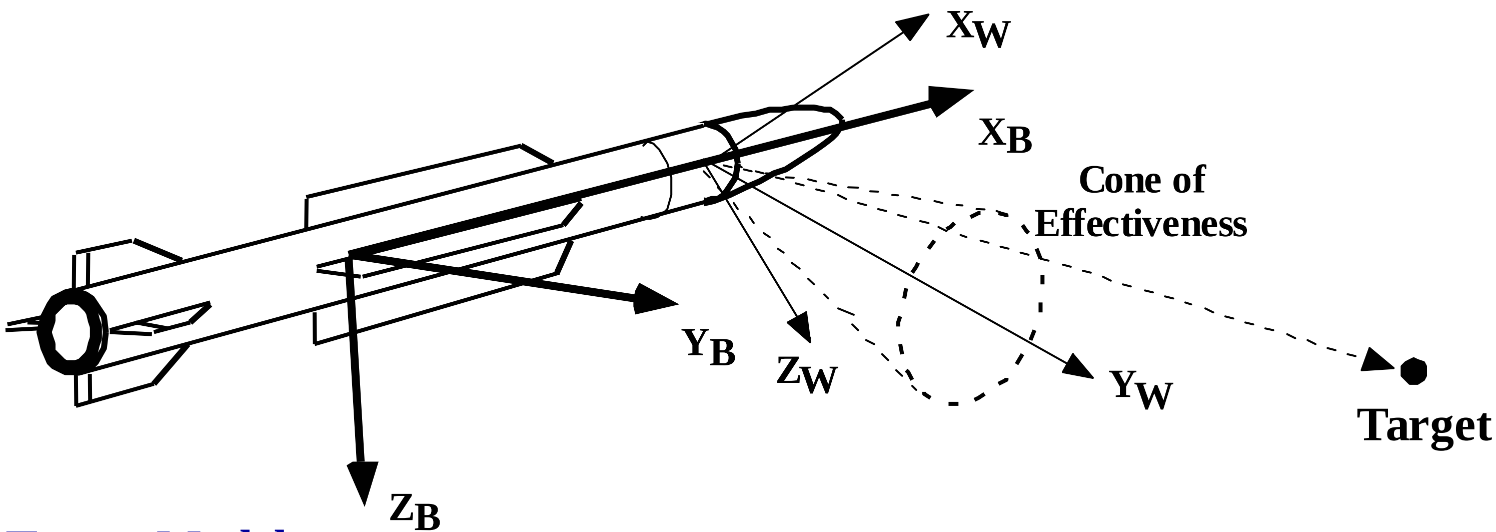
$$\dot{P} - Q(x) - P\,g(x)\,R^{-1}(x)\,g^T(x)\,P < 0$$

Integrated Guidance-Control System Design Philosophies

- **Zero-Effort-Miss Guidance-Control Law**
- **Pure-Pursuit Guidance-Control Law**
- **Proportional Navigation Guidance-Control Law**
- **Finite-Interval Terminal-Miss Minimizing IGC**

Missile and Target Models

• **Six Degree-of-Freedom Air to Air Missile Model, Fixed-Aim Warhead:**



• **Target Model:**

- **Point-Mass Model (F-4 - Like Performance)**

$$\ddot{x}_T^V = \frac{T-D}{m} - g\sin\theta$$

$$\ddot{y}_T^V = a_n \sin\phi$$

$$\ddot{z}_T^V = a_n \cos\phi + g\cos\theta$$

$$\begin{bmatrix} \ddot{x}_T^I \\ \ddot{y}_T^I \\ \ddot{z}_T^I \end{bmatrix} = T_{I/V} \begin{bmatrix} \ddot{x}_T^V \\ \ddot{y}_T^V \\ \ddot{z}_T^V \end{bmatrix}$$

• **Straight and Level Flight Or 5g Weave Maneuver at 0.2 Hz.**

Zero-Effort Miss IGCS

- **Zero-Effort Miss (ZEM):** $z = x + \dot{x}\, t_{go}$

  Where: $x = (y_T - y_M)$ $\qquad \dot{x} = (\dot{y}_T - \dot{y}_M)$

  Relative Position and Velocity Vectors

- **Zero-Effort Miss Rate:**

$$\dot{z} = (a_T - a_M)\, t_{go}$$

- **Integrated Guidance-Control System Objective: Drive the Zero-Effort Miss Components Normal to the Missile Y and Z Body Axes to Zero.**

Zero-Effort Miss IGCS

Integrated Guidance-Control System Objectives:

1. Use Roll Fin Deflection to Regulate the Roll Rate and Roll Attitude Near Zero.

2. Use Pitch Fin Deflection to Generate Pitch Rate, and Consequently the Angle of Attack to Drive the Pitch Component of the ZEM to Zero (While Stabilizing the Pitch Axis Dynamics).

3. Use Yaw Fin Deflection to Generate Yaw Rate and Consequently the Angle of Sideslip to Drive the Yaw Component of the ZEM to Zero (While Stabilizing the Yaw Axis Dynamics).

Control Chain:

$$\delta_p \rightarrow p \rightarrow \phi$$

$$\delta_q \rightarrow q \rightarrow \alpha \rightarrow z_{ZEM}$$

$$\delta_r \rightarrow r \rightarrow \beta \rightarrow y_{ZEM}$$

Finite-Interval IGCS Design Using Feedback Linearization

6-DOF Nonlinear Missile Dynamics: (In the Form of a Computer Program)

States: $y, z, \alpha, \beta, \phi, P, Q, R$

Controls: $\delta_p, \delta_q, \delta_r$

Point-Mass Target Dynamics (Open-Loop Maneuvers):

States: x, y, z

Controls: a_y, a_z

↓

Feedback Linearize the System Dynamics In Terms Of Target Relative Missile Position Components and Their Derivatives (Numerically Carried Out Using the Nonlinear Synthesis Tools™ Software)

Finite-Interval IGC System Design Using Feedback Linearization

• **Feedback Linearized Missile-Target Dynamics:**

$$\ddot{z}_c = f_1(.) + g_{11}(.)\delta_p + g_{12}(.)\delta_q + g_{13}(.)\delta_r \equiv v_1$$
$$\ddot{y}_c = f_2(.) + g_{21}(.)\delta_p + g_{22}(.)\delta_q + g_{23}(.)\delta_r \equiv v_2$$
$$\ddot{\phi} = f_3(.) + g_{21}(.)\delta_p + g_{22}(.)\delta_q + g_{23}(.)\delta_r \equiv v_3$$

• **Can be Placed in the LTI Form in terms of Transformed States and Controls:**

$$\dot{x} = A\,x + B\,u$$

• **Formulate a Finite-Interval Optimal Control in terms of Transformed States and Controls:**

$$J = \frac{1}{2} x_f^T S_f x_f + \frac{1}{2}\int_{to}^{tf}\left(x^T Q x + u^T R u\right) d\tau$$

Finite-Interval IGC System Design Using Feedback Linearization

- **Solution to the Transformed Optimal Control Problem:**

$$u = -R^{-1}B^{T}\left[\left(S - TV^{-1}T^{T}\right)x + TV^{-1}\psi\right]$$

Jerk, Acceleration, Velocity, Position Optimizing Control

Where: $\dot{S} = -A^{T}S - SA - Q + SBR^{-1}B^{T}S, \quad S(t_f) = S_f$

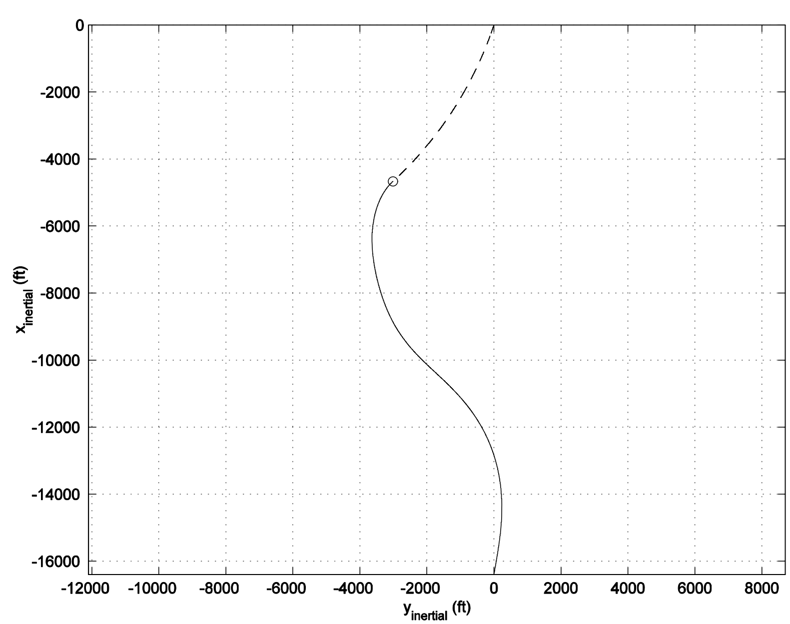
$$\dot{V} = T^{T}BR^{-1}B^{T}T, \quad V(t_f) = [0]_{q\times q}$$

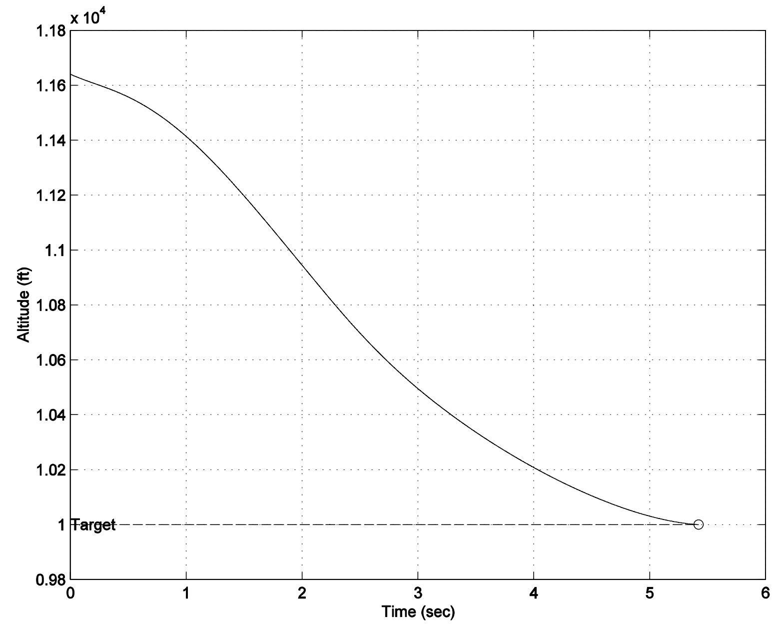
$$\dot{T} = \left(SBR^{-1}B^{T} - A^{T}\right)T, \quad T(t_f) = \begin{bmatrix} I_q \\ [0]_{(n-q)\times q} \end{bmatrix}$$

Pseudo Control Vector can be Inverse Transformed to Obtain Fin Deflections

Engagement Scenario

- Missile at 11,640 ft & Target at 10,000 ft Altitude.
- Target Speed 1,037 ft/s, Heading 198.4 degrees, Maneuver Acceleration 3g.

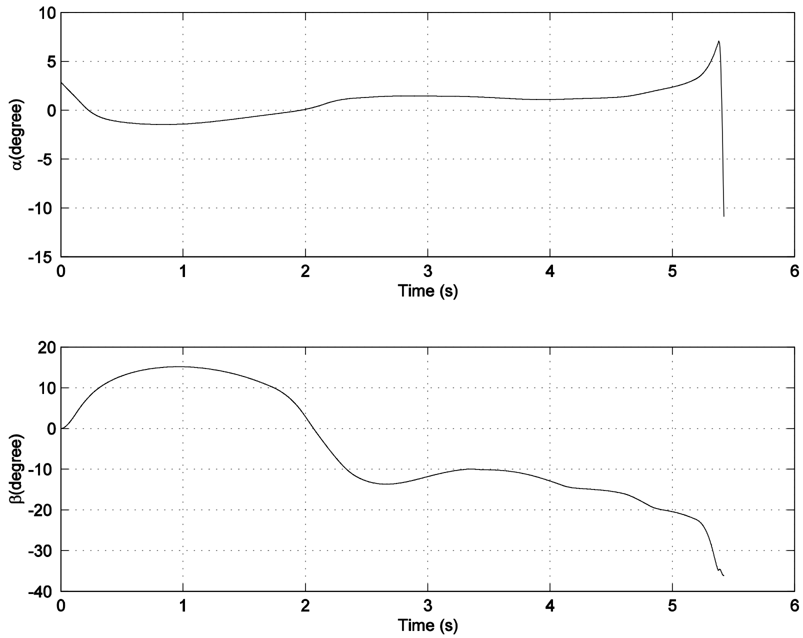


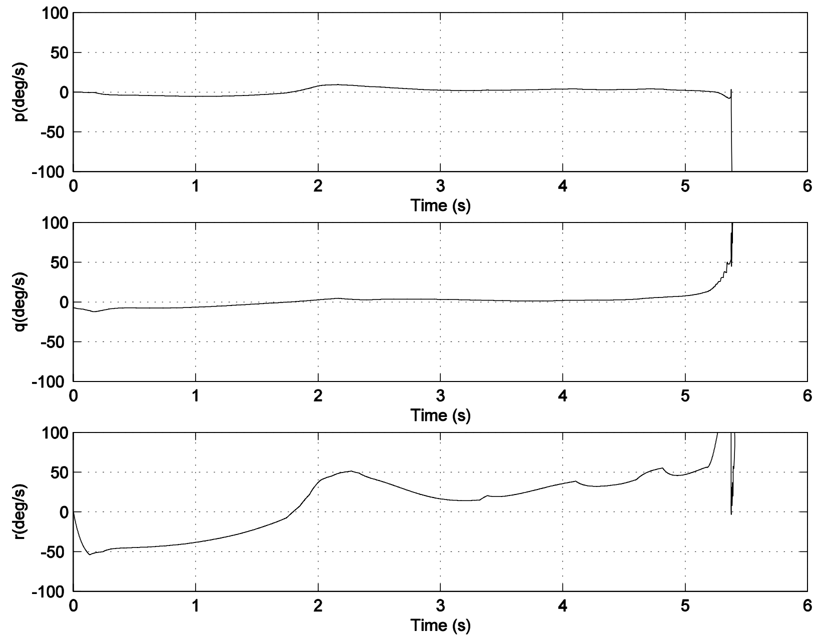


Horizontal Plane Trajectories **Vertical Plane Trajectories**

Miss Distance: 0.035 ft

Engagement Scenario

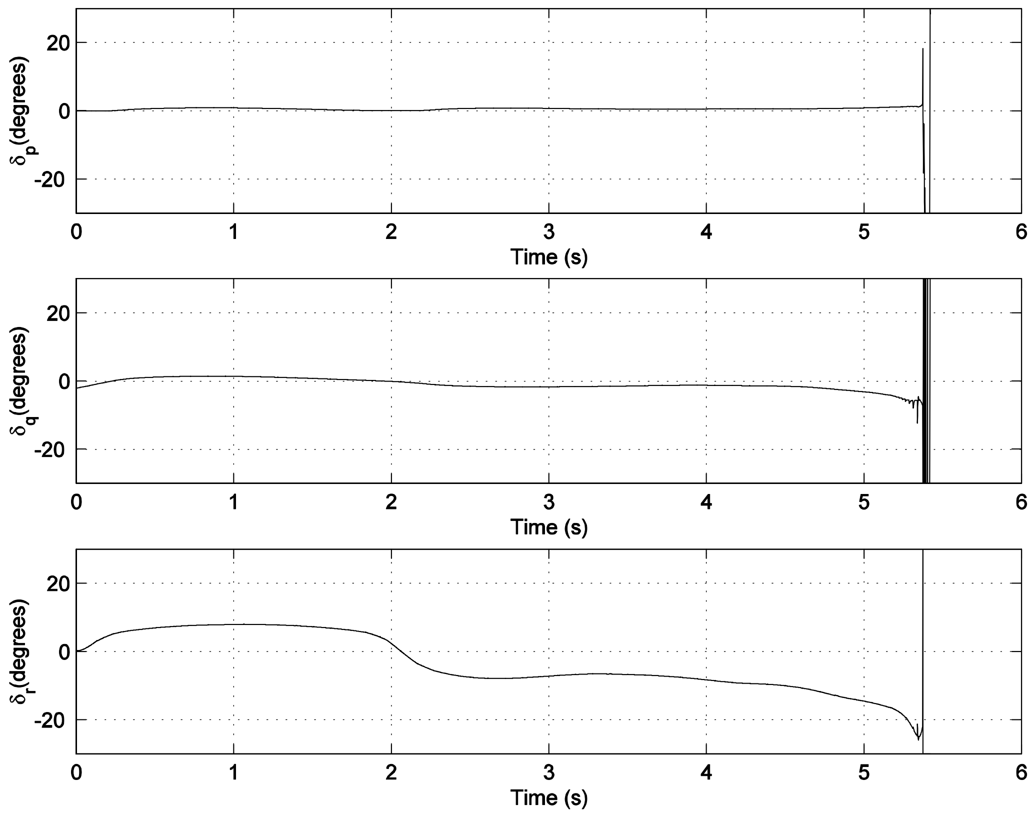




α, β Histories

P, Q, R Histories

Engagement Scenario



Fin Deflection Time Histories

KKV Guidance-Control

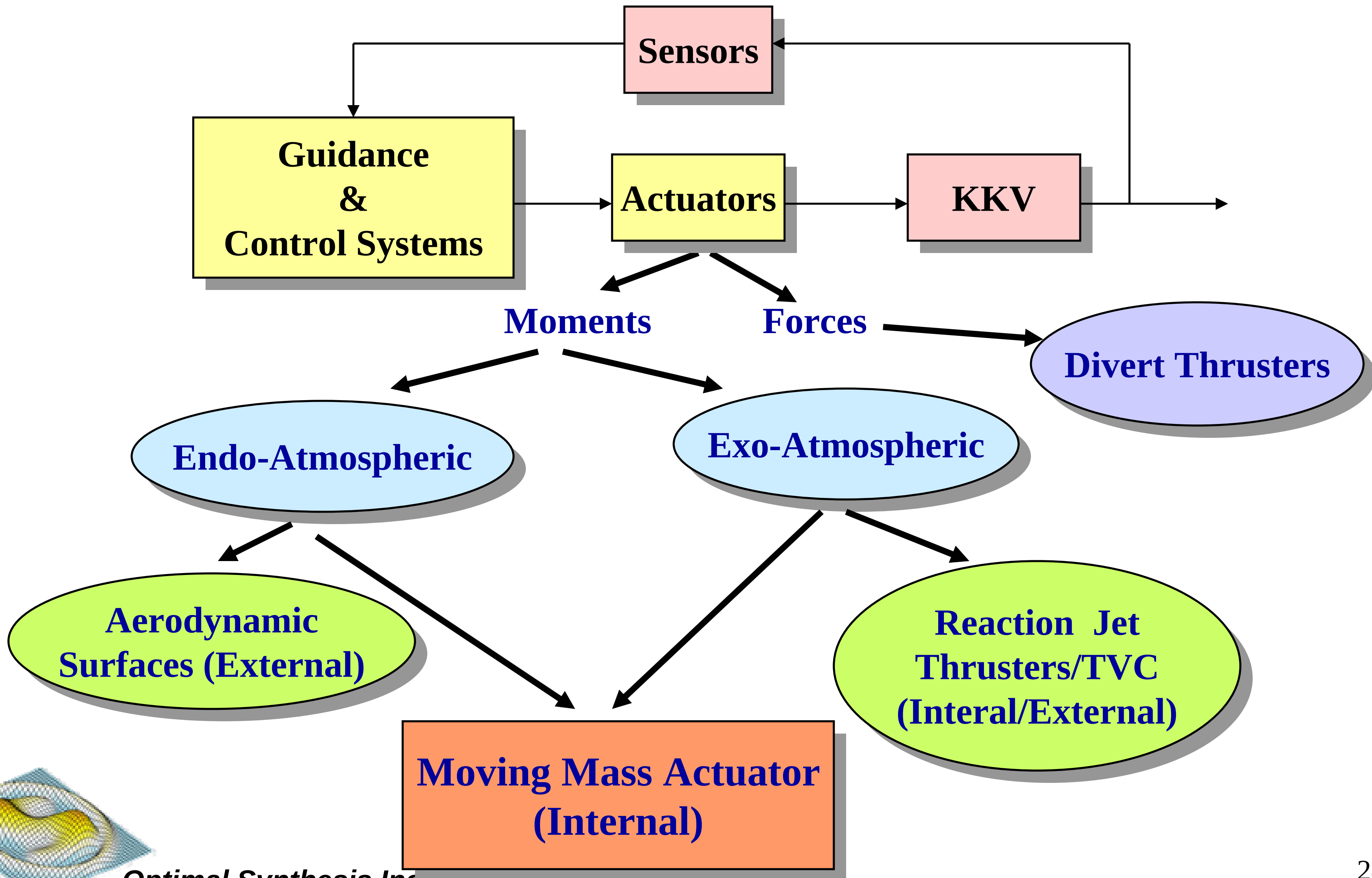
- **Agile Guidance and Control System Necessary for Hit-to-kill**
- **Must Exploit System Nonlinearities to Enhance Performance**
- **Integrated Design of Guidance-Control Systems Using Feedback Linearized Dynamics and,**

  - *LOS* Rate Regulation

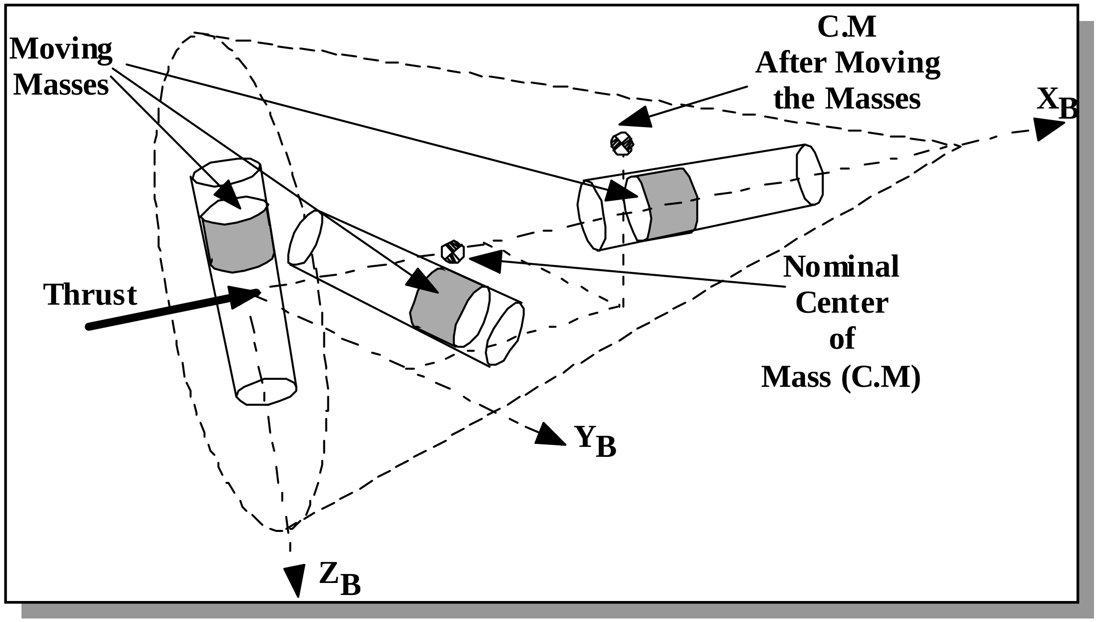
Background Work:

- **Software for Nonlinear Control System Design: *Nonlinear Synthesis Tools™* (1997-2000).**
- **Missile Integrated Guidance-Control System Design Methodologies (1999 - 2002)**

KKV IGCS - Actuation Options



Moving-Mass Actuators for *KKV*



Principle of Operation: **Moving Masses Alter the CG Location of the *KW*, thereby Creating Control Moments from External Forces**

Characteristics of Internal Actuators

<u>Advantages:</u>

1. Effective in any Speed Range (Endo and Exo-Atmospheric Flight)

2. Contained within the *KKV* Geometric Envelope.

3. No Mass Expulsion

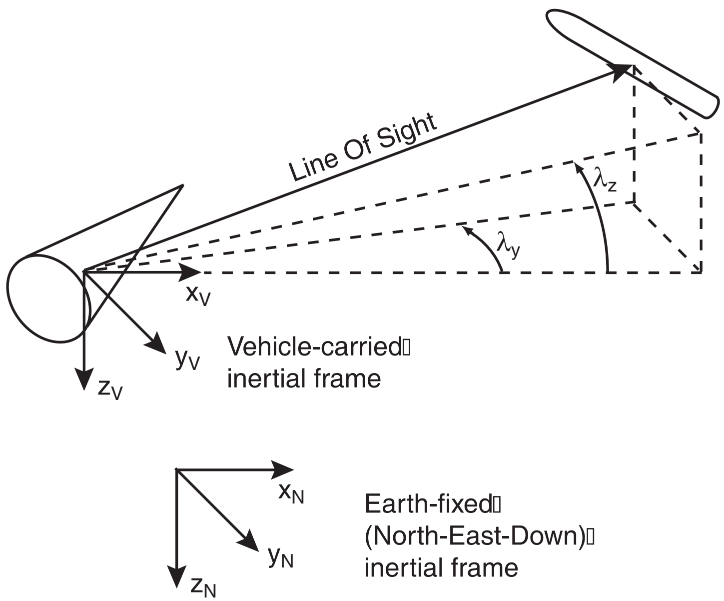
<u>Disadvantages:</u>

Complex, Nonlinear System Dynamics

- **Potential Non-minimum Phase Behavior**
- **Variable Speed of Response**
- **Atmospheric Performance can be Sensitive to *Center of Pressure* Uncertainty.**

Engagement Geometry

Target Model: $\ddot{x} = 0, \quad \ddot{y} = 0, \quad \ddot{z} = g \quad OR \quad \ddot{z} = A\,\omega^2 \sin\omega t$



Line-of-Sight Angles:

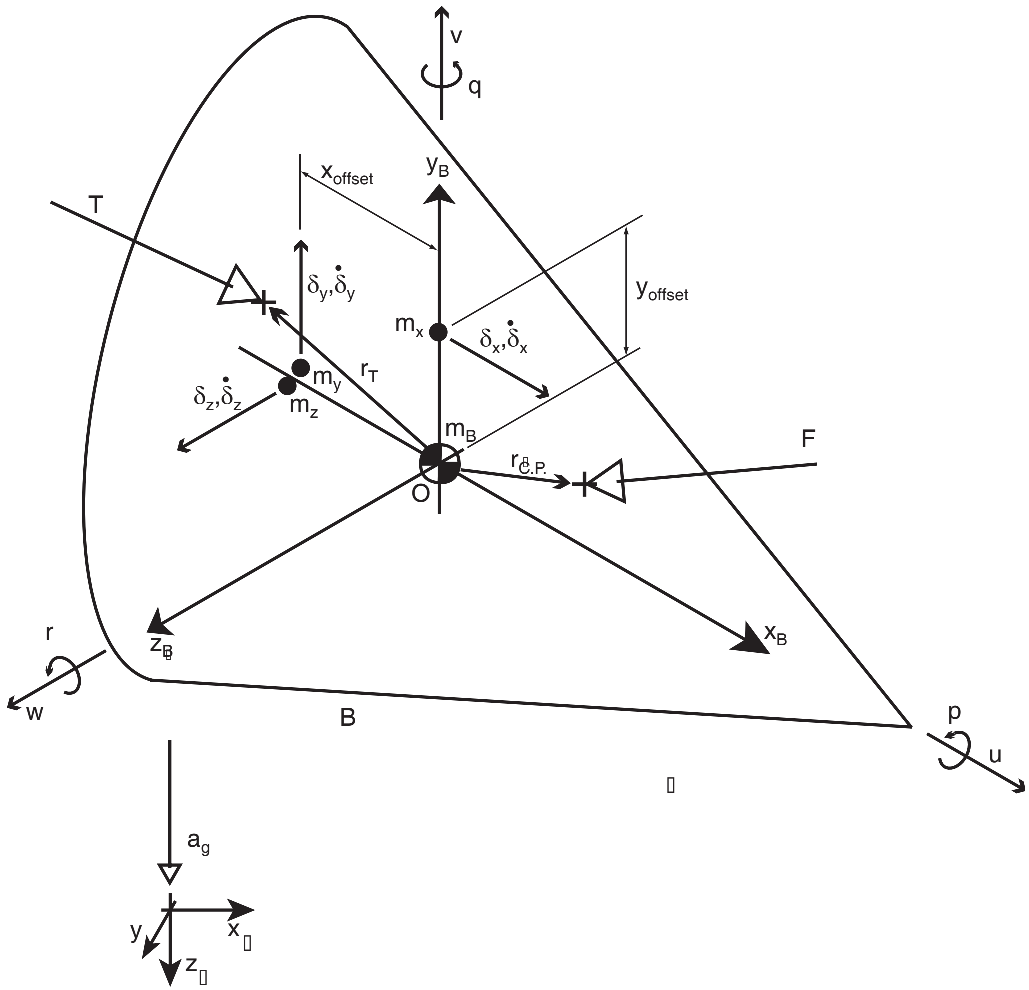
$$\lambda_y = tan^{-1}\left(\frac{r_y}{r_x}\right) \qquad \lambda_z = tan^{-1}\left(\frac{r_z}{r_x}\right)$$

$$\vec{r} = \begin{bmatrix} r_x & r_y & r_z \end{bmatrix}^T \qquad r = \left(r_x^2 + r_y^2 + r_z^2\right)^{1/2}$$

Line-of-Sight Rates:

$$\dot{\lambda}_y = \frac{\dot{r}_y - \dot{r}_x \tan\lambda_y}{r_x \sec^2\lambda_y} = \frac{r_x\dot{r}_y - \dot{r}_x r_y}{r^2} \qquad \dot{\lambda}_z = \frac{\dot{r}_z - \dot{r}_x \tan\lambda_z}{r_x \sec^2\lambda_z} = \frac{r_x\dot{r}_z - \dot{r}_x r_z}{r^2}$$

KKV Dynamics



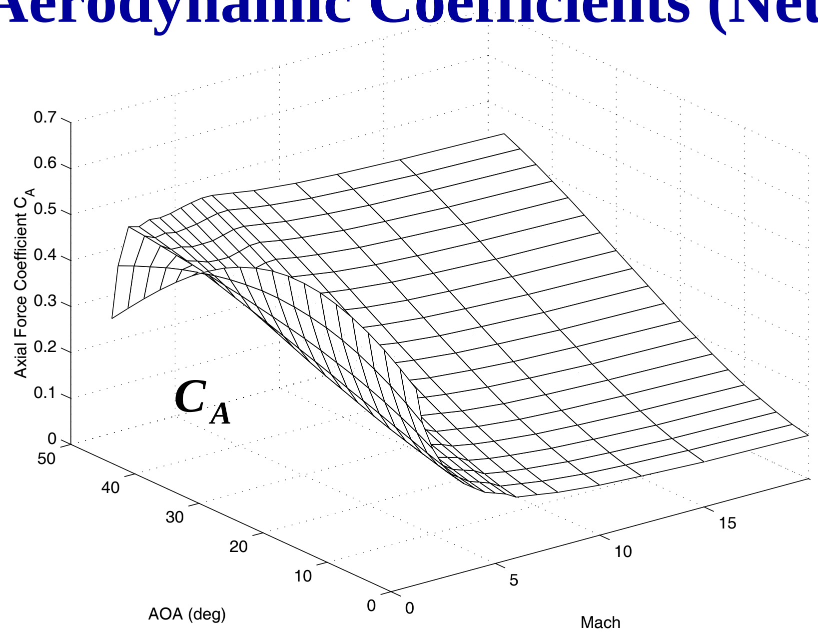
Free-body Diagram of the Moving-Mass *KW*

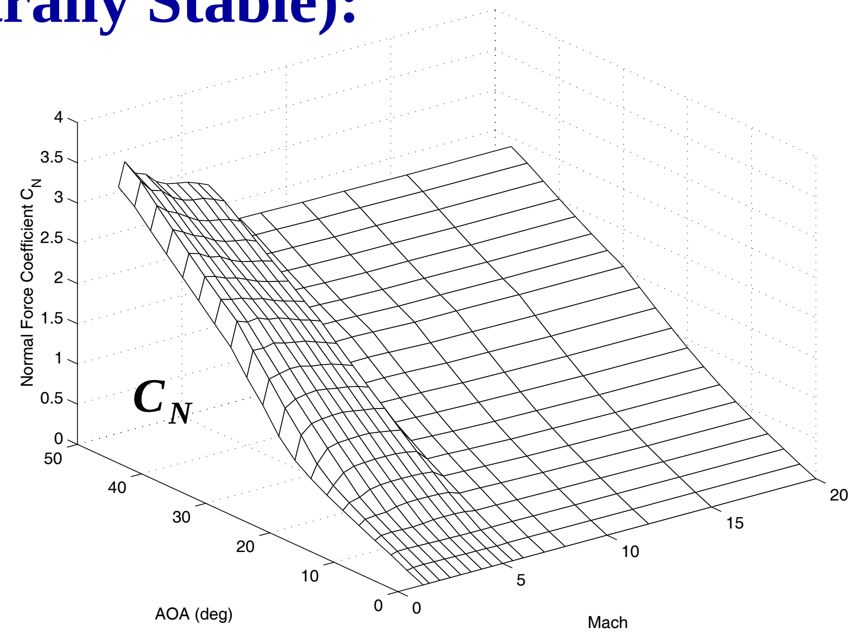
KKV Dynamics

First-Order Moving-Mass Positioning Actuator Dynamics

***KW* Mass: 55 lbs**
***KW* Diameter: 1 ft.**
Actuator Masses: 5 lbs Each

Aerodynamic Coefficients (Neutrally Stable):





Integrated Guidance-Control by *LOS* Rate Regulation

IGC Philosophy:

1. Apply Force u_z on the Pitch Moving Mass to Position it at δ_z along the z-body axis, so as to Generate Pitch Rate q and Consequently a Lateral Velocity w to Drive the Pitch Component of the *LOS* Rate to Zero (While Stabilizing the Pitch Axis Dynamics).

2. Apply Force u_y on the Yaw Moving Mass to Position it at δ_z along the z-body axis, so as to Generate Yaw Rate r and Consequently a Lateral Velocity v to Drive the Yaw Component of the *LOS* Rate to Zero (While Stabilizing the Yaw Axis Dynamics).

Integrated Guidance-Control by *LOS* Rate Regulation

Design Methodology:

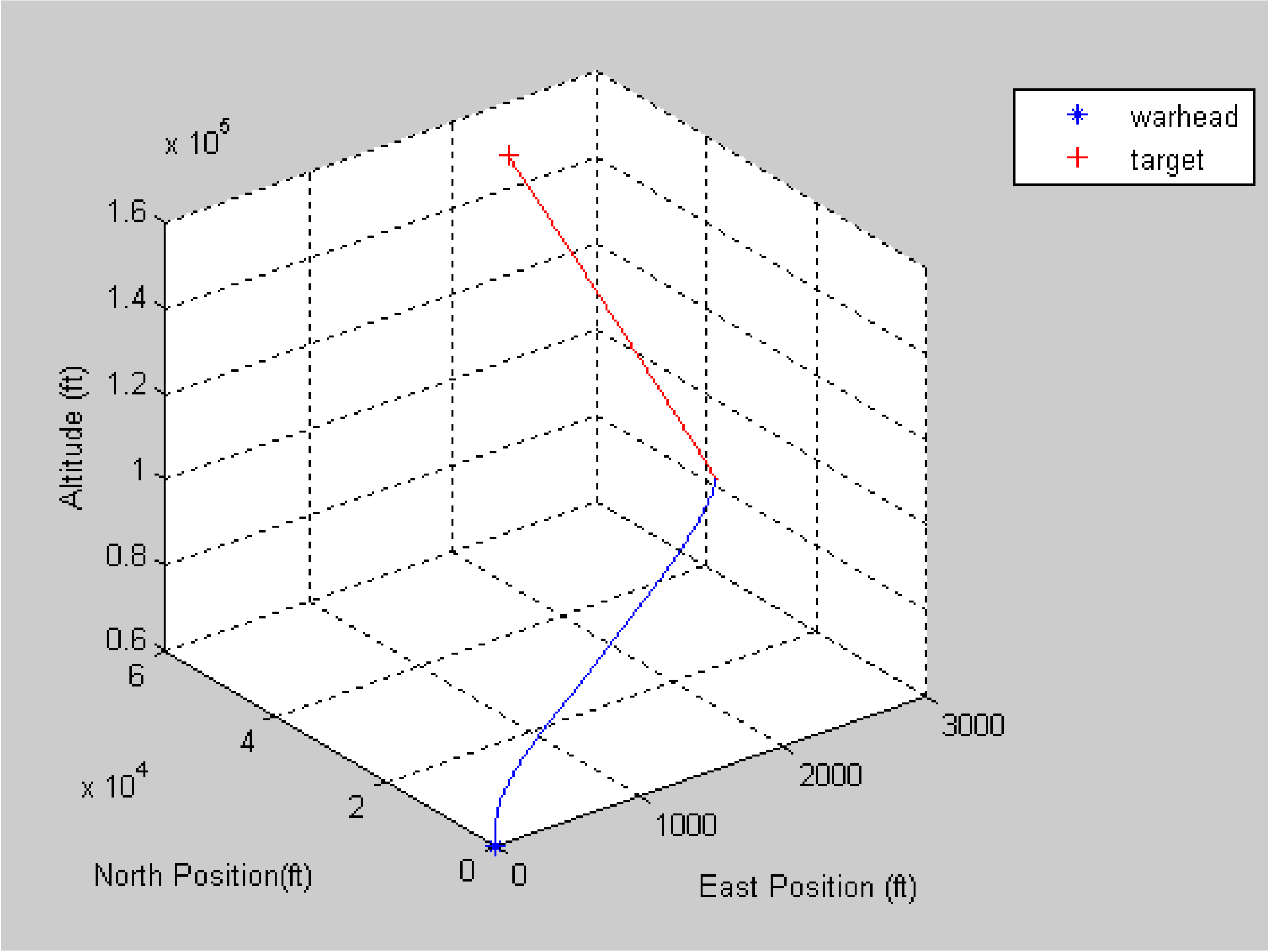
•**Define the Control Chain:**

$$u_z \rightarrow \dot{\delta}_z \rightarrow \delta_z \rightarrow q \rightarrow w \rightarrow \dot{\lambda}_z$$
$$u_y \rightarrow \dot{\delta}_y \rightarrow \delta_y \rightarrow r \rightarrow v \rightarrow \dot{\lambda}_y$$

• **Specify Pole Locations or LQR Design Weights:**
  - **Pole Locations :** ***{-51, -50, -35, -30, -20}***

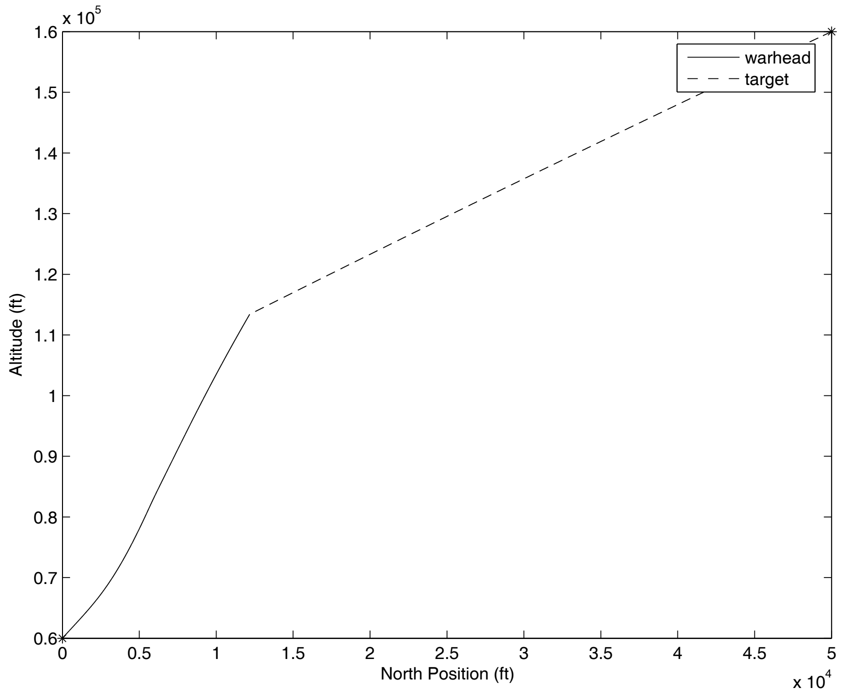
• **Evaluated in Several Engagement Scenarios (Single Nonlinear Design):**
  - **Endo-Exo Atmospheric Interception**
  - **Spiraling Target**

Engagement Scenario 1

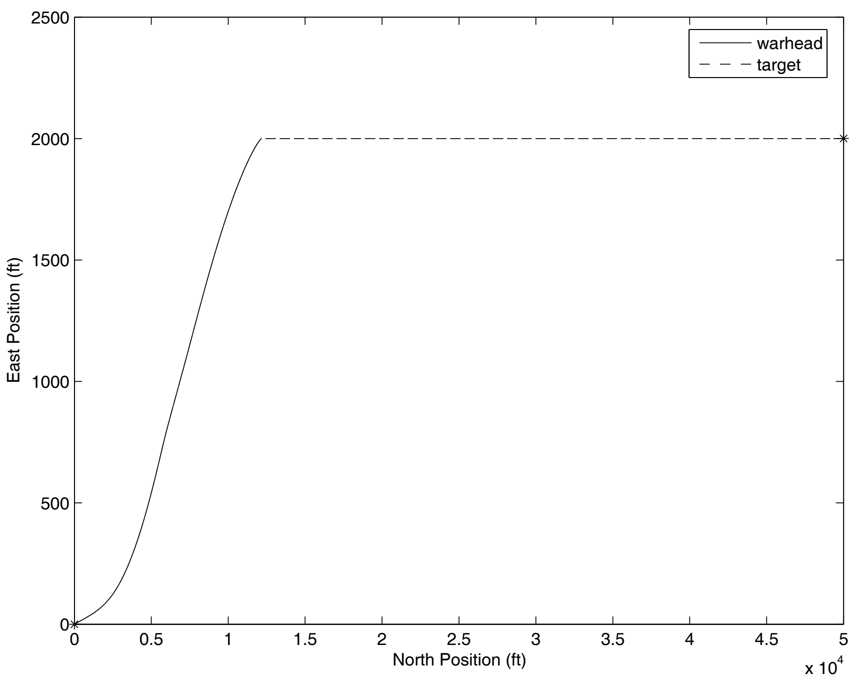


Miss Distance: 0.17 ft

Engagement Scenario 1



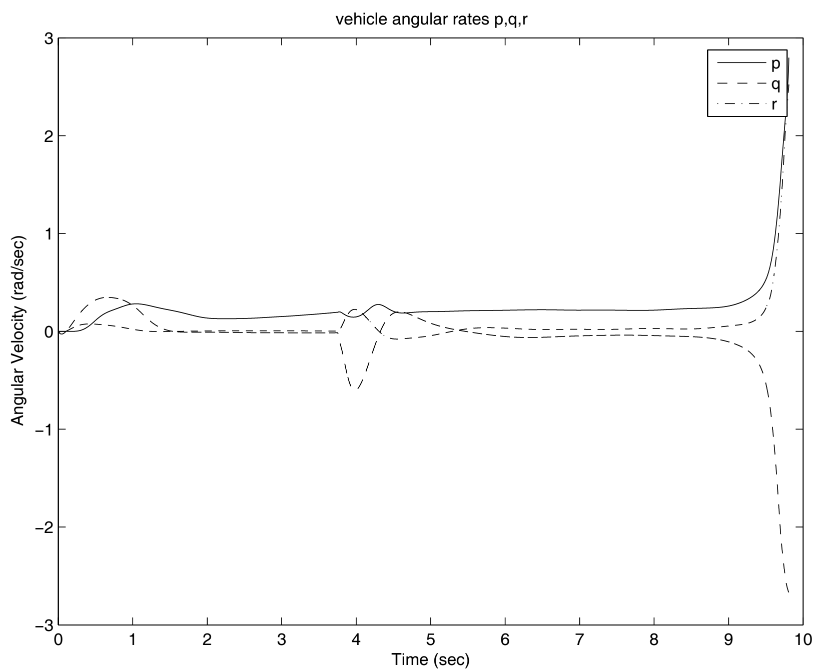
Vertical Plane Trajectories



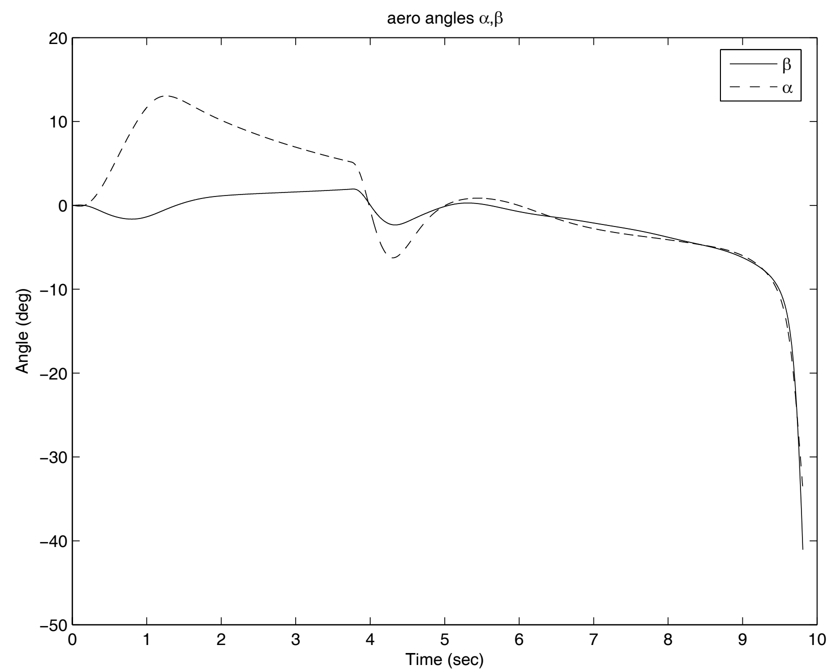
Horizontal Plane Trajectories

Optimal Synthesis Inc.

Engagement Scenario 1

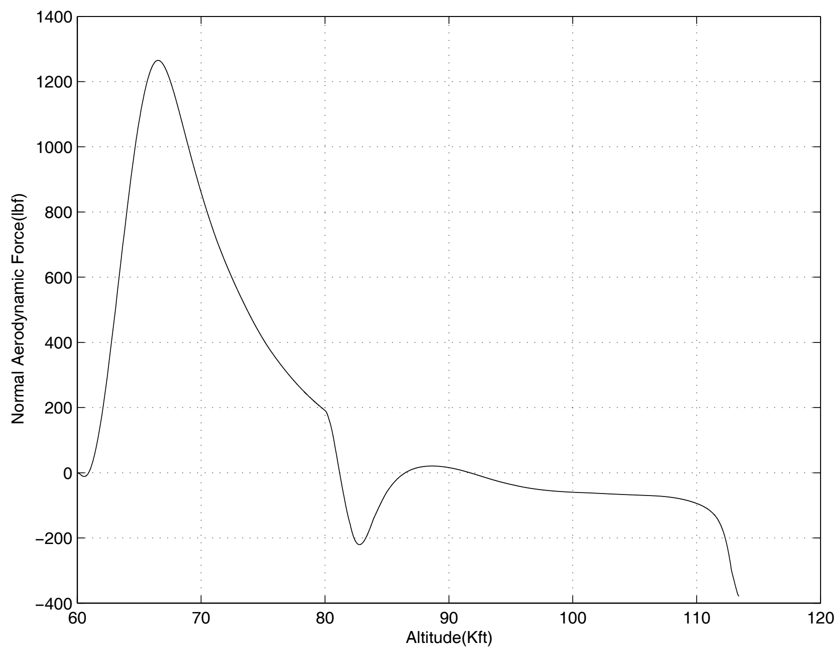


Body Rate Histories

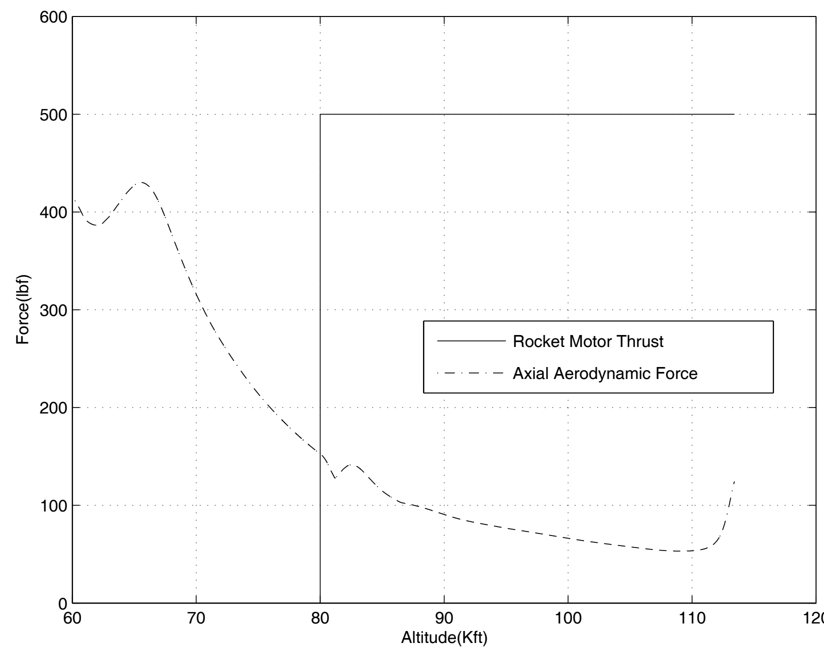


α, β Histories

Engagement Scenario 1

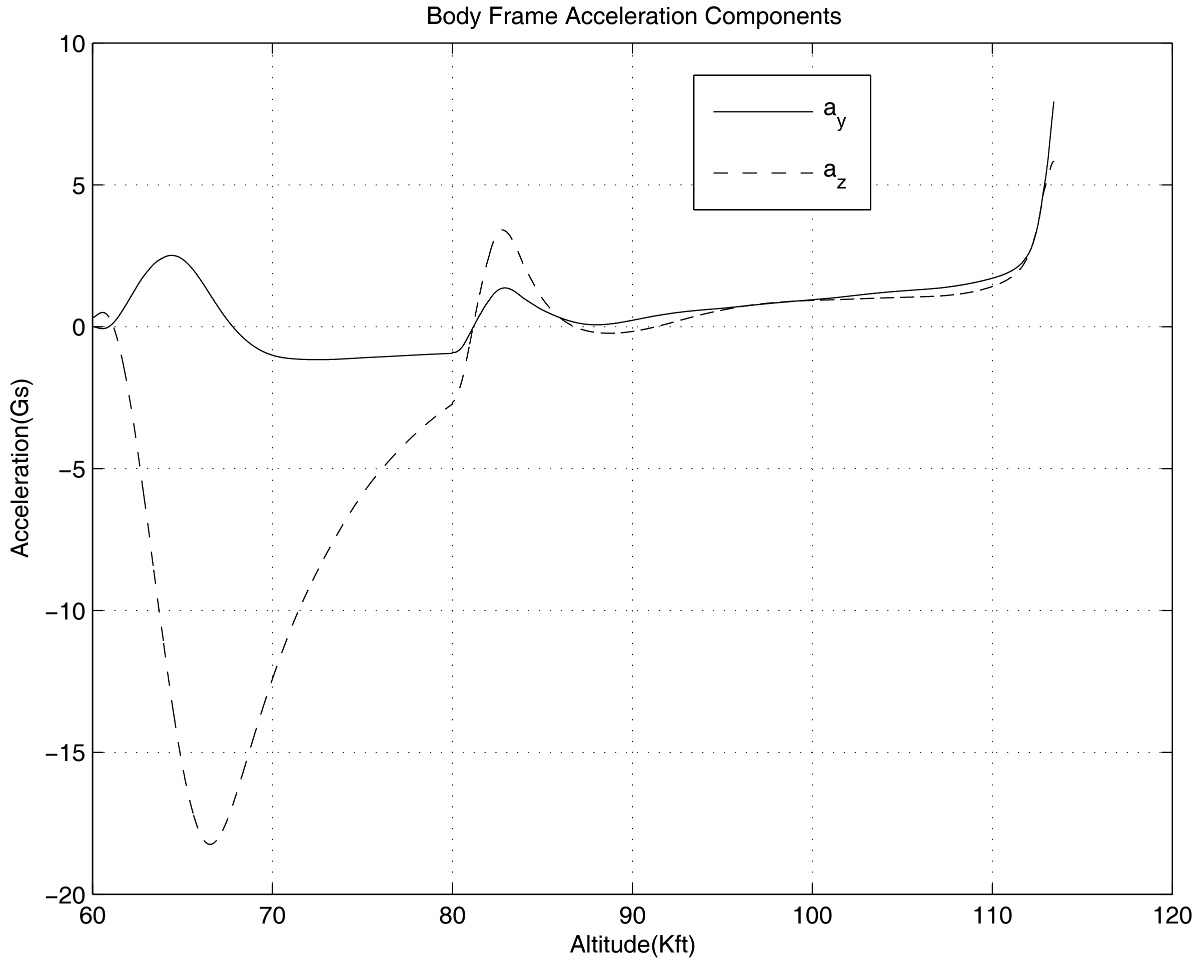


Normal Force History



Axial Force History

Engagement Scenario 1

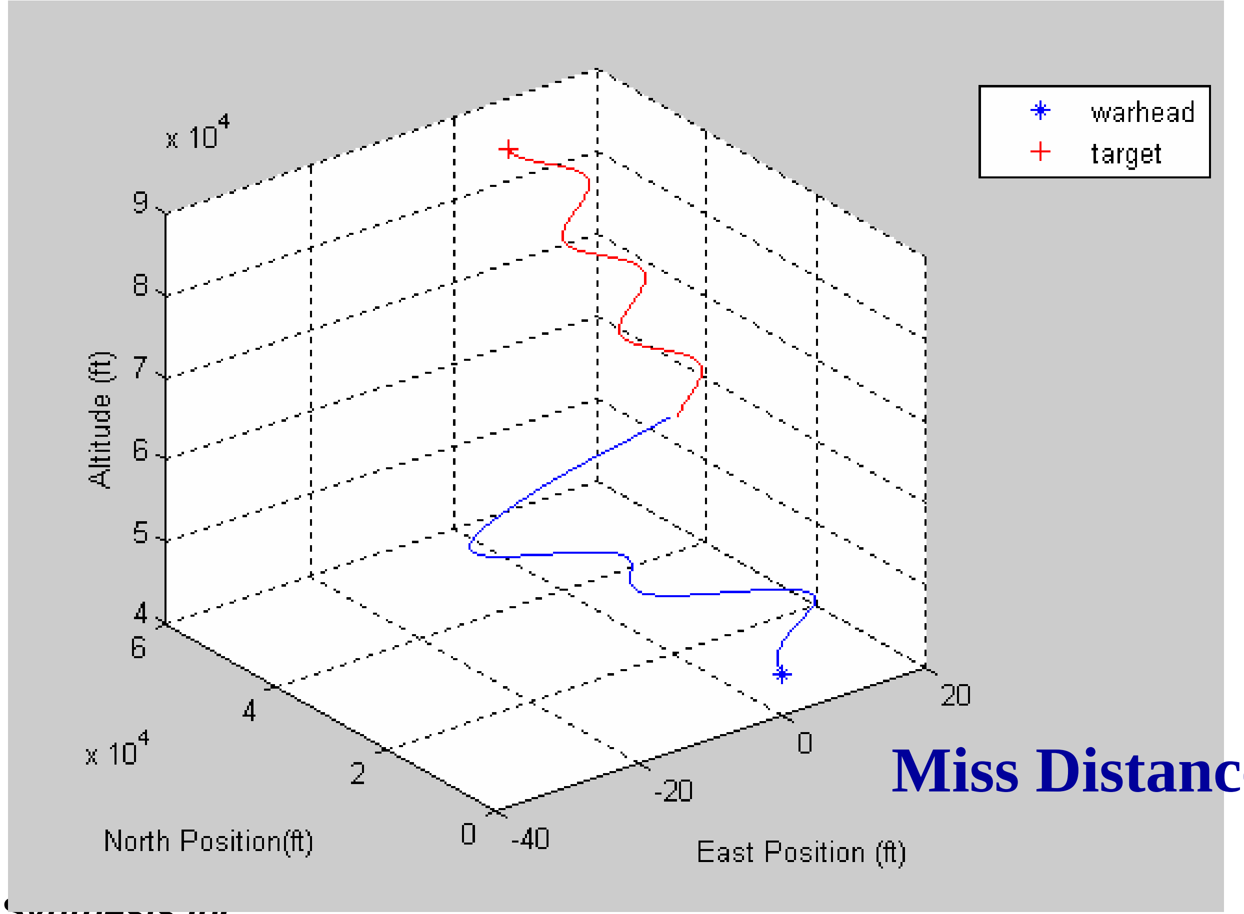


Body Acceleration Histories

Interception of a Spiraling Target

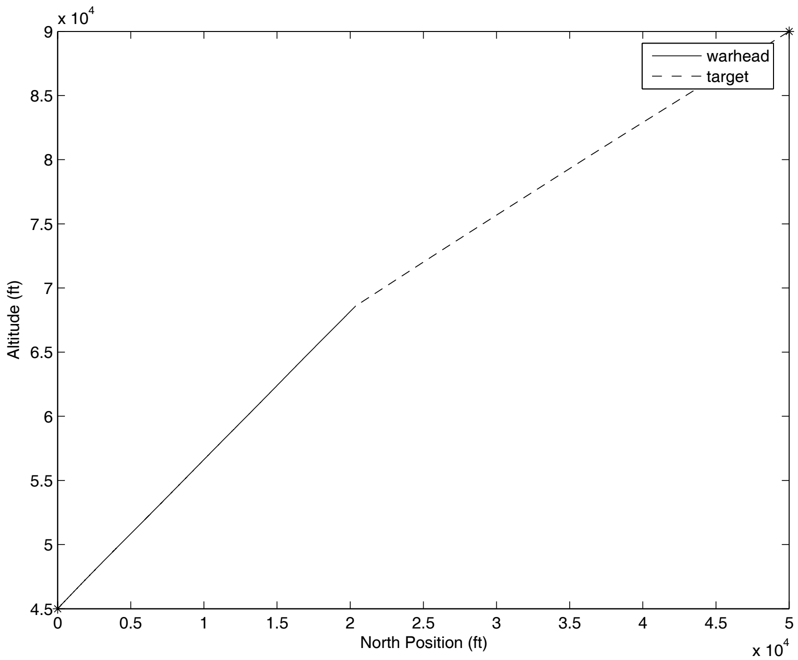
KKV: 45 Kft Altitude , 50 deg Flight Path Angle

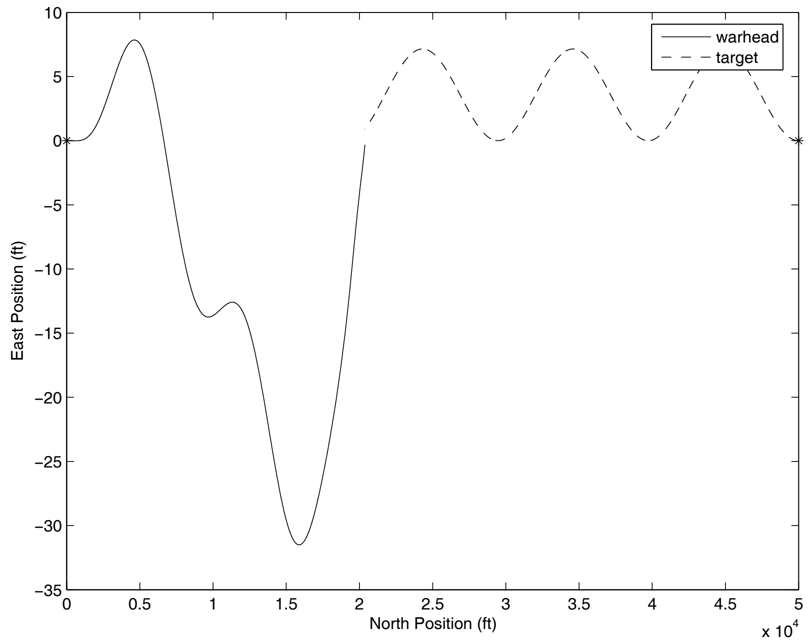
Target: 90 Kft Altitude, 50Kft North, -35 deg Flight Path Angle, 1g Lateral Acceleration, 3 Rad/s Frequency, Reciprocal Heading



Miss Distance: 0.25 ft

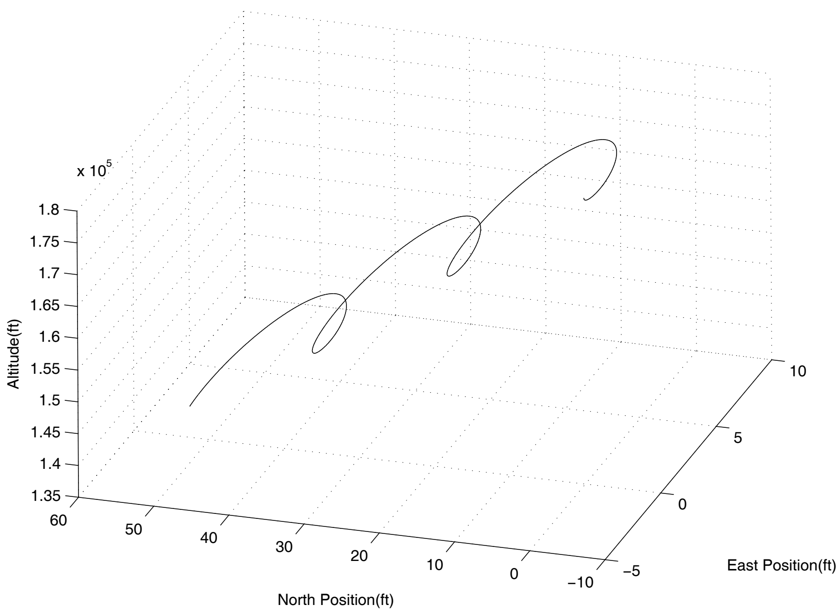
Interception of a Spiraling Target



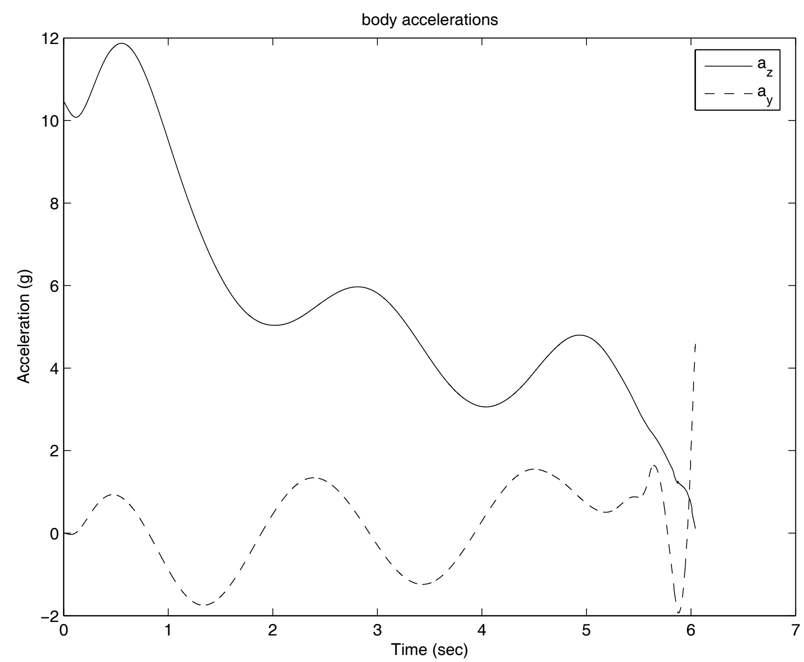


Trajectories in the Horizontal and Vertical Plane

Interception of a Spiraling Target



Target Trajectory After Removing the Secular Component



Pitch-Yaw Acceleration Histories

Real-Time Simulation

Summary

- **Discussed Integrated Guidance-Control System Design Methods**
    - **- Eliminates Iterative Design Process**
    - **- Eliminates Spurious Effects Induced by more Traditional Design Methods**
    - **- Better Satisfaction of Design Objectives**
    - **- Synergistic Design**

- **Illustrated IGC Design for Air-to-Air Missile and an Internally Actuated KKV.**
    - **- Nonlinear Control Techniques**
    - **- Computer-Aided Nonlinear System Design**
    - **- ZEM, LOS Rate, Terminal-Miss Design Strategies**

- **Being Investigated for Designing Flight Control Systems of other Vehicles.**